CSE 312A: Foundations of Computing II
Assignment #2
January 10
due: January 17, at noon.

Instructions:

- **Answers:** When asked for a short answer (such as a single number), also *show and explain your work* briefly. Simplify your final formula algebraically as much as possible, without using your calculator. Then, if the answer is a number rather than a function of some variables, use a calculator to evaluate it and provide the number. For example, for counting problems, your answer might look like this:

  Answer: $\binom{5}{2} - \binom{4}{2} = 4$.

  Explanation: There are $\binom{5}{2}$ ways to select 2 fingers out of the 5, and $\binom{4}{2}$ of them do not involve the thumb.

  Solutions that do not show enough work may not get full credit.

- **Turn-in:** Do not write your name on your pages (your Gradescope account will identify you to us) and do not include a copy of the exercise's question in what you turn in. You must use Gradescope to upload your homework solutions. You will submit a single PDF file containing your solutions to all the exercises in the homework. Each numbered homework question must be answered on its own page (or pages). You must follow the Gradescope prompts that have you link exercise numbers to your pages. You may typeset your solutions on a computer (see here for tutorials and templates) or you can handwrite them, take a picture of (or scan) each handwritten page, and convert the pictures into a single PDF file. You are responsible for making sure that your solution is easily readable, and submitted on time.

1. An airport has the ability to allow 2 aircraft to take off every 10 minutes. In a particular hour, there are 12 aircraft, $A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12$ waiting to take off. They need to be scheduled in the 6 available time slots. However, maintenance constraints mean that 5 of the aircraft: $A1, A2, A3, A4, A5$ must take off in different slots, and must take off in that order. How many possible ways are there to schedule the 12 aircraft to the 6 slots?

2. A Quidditch team (three Chasers, two Beaters, one Keeper, and one Seeker) consisting of 2 boys and 5 girls is to be chosen from a class of 10 boys and 12 girls. If you choose a particular 7 students, changing their team positions is considered to change the team. But a Chaser is a Chaser and a Beater is a Beater, so swapping the two Beaters, for example, does not change the team.

   (a) How many different teams are possible if each child can play any position?

   (b) How many different teams are possible if 4 of the 12 girls can only be Seekers?

   (c) How many different teams are possible if each child can play any position, but Harry, Katie, Angelina, and Alicia cannot all be chosen because, you know, it just wouldn't be fair. (Harry is a boy, and Katie, Angelina, and Alicia are girls.)

3. Santa Claus has 20 identical reindeer treats that he wants to distribute to his 8 reindeer: Dasher, Dancer, Prancer, Vixen, Comet, Cupid, Donner and Blitzen. How many ways can he do this if…

(a) ...every treat must be given to some reindeer? Hint: This is like counting the number of non-negative integer solutions to an equation, a topic we discussed in class.

(b) ...every reindeer must get at least 2 treats, and every treat must be given to some reindeer?

(c) ...some treats may be left over, and some reindeer may get 0 treats?

(d) ...all treats must be distributed, but no reindeer can get more than 5 treats? Hint: use inclusion-exclusion.

4. 10 players enter a tennis tournament. The organizers want to match up the 10 players into 5 groups of 2 to play each other in the first round. How many ways are there to pair the players if ...

   (a) ...there are no further restrictions?

   (b) ...5 of the 10 players are ranked higher (top seeds) than the other 5, and the high-ranked players must be paired with low rank players?

   (c) ...3 of the 10 are top seeds and must not be paired together in the first round?

5. Prove the identity $\binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}$. Hint: Consider the expansion of $(1 + x)^{2n}$, and compute the coefficient of x^n in this expansion using the binomial theorem. Then compute the same coefficient by writing $(1 + x)^{2n} = ((1 + x)^n)^2$ and applying the binomial theorem to $(1 + x)^n$. There is also a combinatorial proof that you can try to find by thinking about picking n elements from $[2n]$ by picking k elements from $[n]$ and $n - k$ elements from $[2n] - [n]$. Any proof is acceptable for full credit.

6. 3 couples go to dinner and are seated at a circular table. Two different seatings of the 6 people are considered equivalent if one can be rotated to give another. How many ways can the couples be seated at the table if...

   (a) ...there are no other restrictions?

   (b) ...no one sits next to their partner on either side? Hint: use the inclusion-exclusion principle.