CSE 312A: Foundations of Computing II
Assignment #1
January 4
due: January 10, at noon.

Instructions:

- **Answers:** When asked for a short answer (such as a single number), also *show and explain your work* briefly. Simplify your final formula algebraically as much as possible, without using your calculator. Then, if the answer is a number rather than a function of some variables, use a calculator to evaluate it and provide the number. For example, for counting problems, your answer might look like this:

  Answer: $\binom{5}{2} - \binom{4}{2} = 4$.

  Explanation: There are $\binom{5}{2}$ ways to select 2 fingers out of the 5, and $\binom{4}{2}$ of them do not involve the thumb.

  Solutions that do not show enough work may not get full credit.

- **Turn-in:** Do not write your name on your pages (your Gradescope account will identify you to us) and do not include a copy of the exercise's question in what you turn in. You must use Gradescope to upload your homework solutions. You will submit a single PDF file containing your solutions to all the exercises in the homework. Each numbered homework question must be answered on its own page (or pages). You must follow the Gradescope prompts that have you link exercise numbers to your pages. You may typeset your solutions on a computer (see here for tutorials and templates) or you can handwrite them, take a picture of (or scan) each handwritten page, and convert the pictures into a single PDF file. You are responsible for making sure that your solution is easily readable, and submitted on time.

1. Did you read and follow the guidelines described above?

2. The English alphabet has 21 consonants and 5 vowels. How many 6 letter words can be made from these letters if . . .

   (a) . . . all letters must be vowels?

   (b) . . . all letters in odd positions must be consonants, and all other letters can be arbitrary?

   (c) . . . all letters in odd positions must be consonants and all letters in even positions must be vowels? (the first letter is considered to be in odd position)

   (d) . . . there must be at least one vowel?

3. Suppose D, R are sets of sizes $|D| = d$, $|R| = r$. How many functions $f : D \to R$ are there if . . .

   (a) . . . there are no further restrictions?

   (b) . . . $r \geq d$ and f must be injective?

   (c) . . . $r = d$ and f must be a bijection?

   (d) . . . $d \geq r = 2$ and f must be surjective?

4. How many pairs of subsets $A, B \subseteq [n]$ have the property that . . .

(a) $\ldots A \cup B = [n]$?

(b) $\ldots A \subseteq B$?

(c) $\ldots A \subseteq B$ and $A \neq B$?

5. Suppose you have the ace, king, and queen of each of the four suits in a single stack of cards. How many arrangements of the cards are there in the stack if …

   (a) …only the rank matters? For example, swapping the positions of two aces is considered the same arrangement.

   (b) …only the suit matters? For example, swapping the positions of two clubs is considered the same arrangement.

6. Suppose Anup is trying to assign the 6 TAs of $312A$ to the 4 sections of the class. How many ways are there to do this if …

   (a) …there are no other restrictions? Here some sections may get no TAs.

   (b) …the first and the last section must be assigned at least one TA, but the other two may be assigned any number of TAs?

   (c) …each section must get at least 1 TA, but at most 4 TAs?

7. Suppose we have a one-dimensional integer array A of size n, where $n \geq 4$. We insert the integers $0, 1, 2, \ldots, n-1$ into this array, with each integer appearing exactly once in the array. We will say that *i is at its own index* if $A[i] = i$. Give a very simple formula (e.g., no summation in your final answer) for each of the following. How many arrangements are there in which …

   (a) at least 1 entry is not at its own index?

   (b) at least 2 entries are not at their own indices?

   (c) at least 3 entries are not at their own indices?

   (d) at least 4 entries are not at their own indices?