CSE 312: Foundations of Computing II

Section 10: Final Review

0. Sorted Hands

A bridge deck consists of 52 cards divided into 4 suits of 13 ranks each. A bridge hand consists of 13 cards from a bridge deck. Suppose that the bridge cards are well shuffled and dealt. What is the probability that your bridge hand is already sorted when you pick it up, given that you have been dealt at least two cards in each of the 4 suits? By "sorted" I mean that the cards of any one suit are adjacent to each other, and the cards of each suit are sorted by rank, with ascending ranks either from left to right or from right to left in your hand. The 4 suits can be in any order in your hand, and different suits can sorted in different directions.

1. Vampire City

For any individual x born in Transylvania with a vampire father, there is a 50% chance that x is a vampire, independently for each birth. These are the only conditions under which a new vampire can be created. 75% of the Transylvanian males are vampires. Suppose Igor, a man who has lived in Transylvania his whole life, has three children that are not vampires.

(a) What is the probability that Igor is a vampire?

(b) If Igor has a fourth child, what is the probability that child will be a vampire?

2. "Hey, let's poke your tires for fun!"

Bob is teaching Alice how to play his new favorite game. In each round, Bob shoots an arrow at the tires of Alice's car. He hits with probability p, independent of previous rounds. If he hits a tire, he gets 10 points. If he misses, he loses 5. Let X be Bob's score after n rounds.

(a) What is $\mathbb{E}[X]$?

(b) What is Var(X)?

3. $\frac{1}{\text{A Random Variable}}$

Let X be a continuous random variable with probability density function:

$$f_X(x) = \begin{cases} 2x & \text{, if } 0 \leq x \leq 1 \\ 0 & \text{, otherwise} \end{cases}$$

(a) Find $\mathbb{E}\left[\frac{1}{X}\right]$.

(b) Compute Pr$(X = 0.5)$.

4. POWER SETS

Suppose A and B are random, independent, nonempty subsets of $\{1, 2, \ldots, n\}$, where each nonempty subset is equally likely to be chosen as A or B. What is Pr$(\max(A) = \max(B))$?

5. Gumbel(μ, β)

Suppose $x_1, x_2, \ldots, x_n$ are independent samples from Gumbel(μ, β). A Gumbel distribution, or **Generalized Extreme Value distribution Type-I**, is used to model the distribution of the maximum (or the minimum) of a number of samples of various distributions. The probability density function for a typical Gumbel distribution is

$$f_X(x|\mu, \beta) = \frac{1}{\beta} \exp\left(-\frac{x-\mu}{\beta} - \exp\left(-\frac{x-\mu}{\beta}\right)\right)$$

where $\beta > 0$. Given that we know β, what is the maximum likelihood estimator for μ? Don't forget to prove that it is a maximum of the likelihood function.