# CSE 312: Foundations of Computing II

## Section 9: Concentration Inequalities and Maximum Likelihood

## 0. Get an Inch, Take a MLE

Suppose $x_1, \ldots, x_n$ are iid realizations from density

$$f_X(x \mid \theta) = \begin{cases} \frac{\theta x^{\theta-1}}{3^\theta}, & 0 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

Find the MLE for $\theta$.

## 1. Independent Shreds, You Say?

You are given 100 independent samples $x_1, x_2, \ldots, x_{100}$ from Bernoulli($p$), where $p$ is unknown. These 100 samples sum to 30. You would like to estimate the distribution's parameter $p$. Give all answers to 3 significant digits.

(a) What is the maximum likelihood estimator $\hat{p}$ of $p$?

(b) Is $\hat{p}$ an unbiased estimator of $p$?

## 2. Y Me?

Let $Y_1, Y_2, \ldots Y_n$ be i.i.d. random variables with density function

$$f_Y(y|\sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|y|}{\sigma}\right)$$

.

Find the the MLE for $\sigma$ in terms of $|y_i|$.

## 3. It Means Nothing

(a) Suppose $x_1, x_2, \ldots, x_n$ are samples from a normal distribution whose mean is known to be zero, but whose variance is unknown. What is the maximum likelihood estimator for its variance?

(b) Suppose the mean is known to be $\mu$ but the variance is unknown. How does the maximum likelihood estimator for the variance differ from the maximum likelihood estimator when both mean and variance are unknown?