CSE 312: Foundations of Computing II

QuickCheck: Maximum Likelihood Estimator Solutions (due Thursday, May 24)

0. x^θ

Let $f_X(x|\theta) = \theta x^{\theta-1}$ for $0 \leq x \leq 1$, where θ is any positive real number. Let $x_1, x_2, ..., x_n$ be independent and identically distributed samples from this distribution. Derive and verify the maximum likelihood estimator $\hat{\theta}$.

Solution:

$$L(x_1, x_2, ..., x_n|\theta) = \prod_{i=1}^{n} \theta x_i^{\theta-1}$$

$$\ln L(x_1, x_2, ..., x_n|\theta) = \sum_{i=1}^{n} (\ln\theta + (\theta - 1)\ln x_i)$$

$$\frac{\partial}{\partial\theta} \ln L(x_1, x_2, ..., x_n|\theta) = \sum_{i=1}^{n} (\frac{1}{\theta} + \ln x_i)$$

Set the first derivative to 0,

$$\frac{n}{\hat{\theta}} = -\sum_{i=1}^{n} \ln x_i$$

$$\hat{\theta} = -\frac{n}{\sum_{i=1}^{n} \ln x_i}$$

To verify it's a global maximum,

$$\frac{\partial^2}{\partial\theta^2} \ln L(x_1, x_2, ..., x_n|\theta) = \sum_{i=1}^{n} -\frac{1}{\theta^2} < 0$$

so $\ln L(x_1, x_2, ..., x_n|\theta)$ is concave downward everywhere.