CSE 312: Foundations of Computing II

Section 8: Variance, Important Discrete Distributions

0. What if we lose ?

Suppose 59 percent of voters favor Proposition 600. Use the Normal approximation to estimate the probability that a random sample of 100 voters will contain:

(a) at most 50 in favor. Mention any assumption that you make.

(b) more than 100 voters in favor or less than 0 voters in favor. Will the probability be non zero ?

1. By parts? !

Hopper loves exponentials, so he attempts to make a simple continuous distribution with an exponential. Hopper decides that the distribution will be of the form

$$\begin{cases} a \cdot e^x + b & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

and that the distribution will have a mean equal to 0.5. Can hopper find such a distribution with $a \neq 0$?

2. Driving in Seattle

Yael's driving time to work is between 15 and 20 minutes if the day is sunny, and between 20 and 25 minutes if the day is rainy, with all times being equally likely in each case. Assume that a day is sunny with probability $\frac{1}{4}$ and rainy with probability $\frac{3}{4}$.

(a) What is the PDF of the driving time, viewed as a random variable X?

(b) What is the CDF of the driving time?

(c) What is $\mathbb{E}[X]$? You can leave your answer as an integral.

(d) Find the variance of the driving time in two different ways. You can leave both answers as integrals.

3. "Are we really twins ? " pmf asked pdf

Let $X \sim$ Uniform$(0, 1)$ be a continuous uniform r.v.

(a) Find the CDF of X.

(b) Find the pdf of X^2. (Hint : sometimes, it is easier to calculate the CDF first.)

(c) Was your answer in part(b) a uniform random variable ? If not, try to explain the result.

(d) Let $Y \sim$ Uniform(0, 1) which is independent of X. Use the analogies between continuous and discrete random variables to find Pr$(X < Y)$. (Hint : The analogue of Pr$(Y = k)$ is $p_Y(k) \cdot dy$.)

4. I want views !!! :(

Sharpnel has three videos on youtube called A, B, and C. Video A receives 10 views per month on average, video B receives 15 views per month on average, and video C receives 25 views per month on average. Every view is received independently of any other view.

(a) What is the probability that Sharpnel receives at least a million views next month and becomes very rich ? You can leave your answer as a summation.

(b) What is the probability that Sharpnel only receive views on video A this month ?

(c) What is the expected time Sharpnel has to wait for until he receives the next view?

5. Servers

Your company has 5000 servers. It costs a lot of money to run servers. From last year's data, you know $x_i =$ "money the i^{th} server cost you last year". Assume each server's cost is independent of any other server's cost.

(a) Using the law of large numbers, find an estimator for the expected cost per server.

(b) Using the law of large numbers and the result from part (a), find an estimator for the variance per server. (Hint : Your first instinct is likely to be correct. Think about what you would do if you didn't know about the Law Of Large Numbers)

(c) Using a normal distribution and our results from part(a) and part(b), approximate the probability that the total cost will exceed C. Your answer will not be a number; it will be an expression in terms or ϕ.

6. A square dartboard ?

You throw a dart at an $s \times s$ square dartboard. The goal of this game is to get the dart to land as close to the lower left corner of the dartboard as possible. However, your aim is such that the dart is equally likely to land at any point on the dartboard. Let random variable X be the length of the side of the smallest square B in the lower left corner of the dartboard that contains the point where the dart lands. That is, the lower left corner of B must be the same point as the lower left corner of the dartboard, and the dart lands somewhere along the upper or right edge of B. For X, find the CDF, PDF, $\mathbb{E}[X]$, and Var(X).