CSE 312: Foundations of Computing II

QuickCheck: Continuous RV's and Central Limit Theorem Solutions

0. Paw-some Experiment

Yael spends a lot of her free time hanging out at the Seattle Meowtropolitan cat cafe and decided to make an experiment to find the weights of all of these cats. She found that their weights are normally distributed with a mean of 8.5 lbs and a standard deviation of 2.3 lbs. Let $X \sim \text{Normal}(8.5, 2.3^2)$ be the random variable for this experiment

(a) What is the probability that one randomly selected cat will weigh more than 9.7 lbs?

Solution:

Because X is a normal random variable:
$\Pr(X > 9.7) = \Pr(\frac{X-8.5}{2.3} > \frac{9.7-8.5}{2.3}) = 1 - \Pr(\frac{X-8.5}{2.3} < \frac{9.7-8.5}{2.3})$
$\approx 1 - \Pr(\frac{X-8.5}{2.3} < 0.52) = 1 - \Phi(0.52)$

(b) What is the probability that four randomly-selected cats will have a mean weight of more than 9.7 lbs?

Solution:

Let Y be defined as the average of X.
Since the sum of independent normal random variables is also a normal random variable, $\mu_Y = \mu$ and $\sigma_Y = \frac{\sigma}{\sqrt{4}}$
So, $Y \sim \text{Normal}(8.5, 1.15^2)$

Now, we apply the same approach as part (a):
$\Pr(Y > 9.7) = \Pr(\frac{Y-8.5}{1.15} > \frac{9.7-8.5}{1.15}) = 1 - \Pr(\frac{Y-8.5}{1.15} < \frac{9.7-8.5}{1.15})$
$\approx 1 - \Pr(\frac{Y-8.5}{1.15} < 1.04) = 1 - \Phi(1.04)$

(c) Yael just bought a new hammock for her cat Domino. However, the tag on the hammock says that the maximum allowable weight is 38.8 lbs. If three of Domino's cat friends come over to share the hammock with him, what is the probability they will exceed the maximum allowable weight?

Solution:

For the cats not to exceed the maximum weight, their mean weight would have to be $\frac{38.8}{4} = 9.7$
Now, we're just solving for $\Pr(Y > 9.7) = 1 - \Phi(1.04)$, as found in part (b)