CSE 312: Foundations of Computing II

QuickCheck: Midterm Review (due Thursday, May 3)

Name:

0. I Cast Arcane Missiles!

Suppose you and your friend are dueling each other. He just summoned a minion with two health, and you have three shots of arcane missiles each dealing one damage. Knowing that your friend has 10 health, you decided to use all of the missiles and see if you can defeat his minion. Arcane missiles target randomly between that minion and your friend, but since you don't want to waste any firepower, they will not hit targets that have zero or less health. Let X be a random variable that denotes the number of missiles that hit your friend.

(a) Find the codomain of X and $p_X(k)$, the probability mass function for X.

(b) Find $\mathbb{E}[X]$ by definition of expectation.

(c) Sharpnel insisted on using linearity of expectation to find $\mathbb{E}[X]$. He wrote the following:

Let X_1, X_2, X_3 be random variables that denote the outcome for each arcane missile: 1 if it hit your friend, and 0 if it hit the minion.

$$\mathbb{E}[X_1] = \mathbb{E}[X_2] = \mathbb{E}[X_3] = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 = \frac{1}{2} \tag{1}$$
$$\mathbb{E}[X] = \mathbb{E}[X_1 + X_2 + X_3] \tag{2}$$
$$= \mathbb{E}[X_1] + \mathbb{E}[X_2] + \mathbb{E}[X_3] \tag{3}$$
$$= \frac{3}{2} \tag{4}$$

"And I just broke the Linearity of Expectation!" He yelled in happiness, trying to utilize this incredible result. If he is correct (which shouldn't be), say so, else briefly explain his error.