CSE 312: Foundations of Computing II

Section 5: Variance, Important Discrete Distributions

0. Best Coach Ever!!

You are a hardworking boxer. Your coach tells you that the probability of your winning a boxing match is 0.2 independently of every other match.

(a) How many matches do you expect to fight until you win 10 times ?

(b) You only get to play 12 matches every year. To win a spot in the Annual Boxing Championship, a boxer needs to win at least 10 matches in a year. What is the probability that you will go to the Championship this year?

(c) Let p be your answer to part (b). How many times can you expect to go to the Championship in your 20 year career?

1. Variance of a Product

Let X, Y, Z be independent random variables with means μ_X, μ_Y, μ_Z and variances $\sigma_X^2, \sigma_Y^2, \sigma_Z^2$, respectively. Find $\text{Var}(XY - Z)$.

2. Deception is my last name

Suppose we roll two fair 5-sided dice independently. Let X be the value of the first die, Y be the value of the second die, $Z = X + Y$ be their sum, $U = \min\{X, Y\}$ and $V = \max\{X, Y\}$.

(a) Find $p_U(u)$.

(b) Find $\mathbb{E}[U]$.

(c) Find $\mathbb{E}[Z]$.

(d) Find $\mathbb{E}[UV]$. (Note that U and V are **not** independent. Why not?).

(e) Find $\text{Var}(U + V)$. You may use the fact that, if $W \sim \text{Uniform}(a, b)$ (meaning equally likely to be any integer $\{a, a+1, ..., b\}$), then $\text{Var}(W) = \frac{(b-a)(b-a+2)}{12}$.

3. True or False?

Identify the following statements as true or false (true means always true). Justify your answer.

(a) For any random variable X, we have $\mathbb{E}\left[X^2\right] \geq \mathbb{E}[X]^2$.

(b) Let X, Y be random variables. Then, X and Y are independent if and only if $\mathbb{E}[XY] = \mathbb{E}[X]\,\mathbb{E}[Y]$.

(c) Let $X \sim \text{Binomial}(n, p)$ and $Y \sim \text{Binomial}(m, p)$ be independent. Then, $X + Y \sim \text{Binomial}(n + m, p)$.

(d) Let $X_1, ..., X_{n+1}$ be independent Bernoulli(p) random variables. Then, $\mathbb{E}[\sum_{i=1}^{n} X_i X_{i+1}] = np^2$.

(e) Let $X_1, ..., X_{n+1}$ be independent Bernoulli(p) random variables. Then, $Y = \sum_{i=1}^{n} X_i X_{i+1} \sim \text{Binomial}(n, p^2)$.

(f) If $X \sim \text{Bernoulli}(p)$, then $nX \sim \text{Binomial}(n, p)$.

(g) If $X \sim \text{Binomial}(n, p)$, then $\frac{X}{n} \sim \text{Bernoulli}(p)$.

(h) For any two independent random variables X, Y, we have $\text{Var}(X - Y) = \text{Var}(X) - \text{Var}(Y)$.