CSE 312: Foundations of Computing II

Section 2: More Combinatorics, Intro Probability (Extra)

0. Fleas on Squares

25 fleas sit on a 5×5 checkerboard, one per square. At the stroke of noon, all jump across an edge (not a corner) of their square to an adjacent square. At least two must end up in the same square. Why?

1. PigONEholes

Let $k \geq 2$ be some integer. Show that there exists a positive integer n consisting of only digits $0, 1$ and no larger than 10^{k+2} such that $k|n$. (Hint: Consider the sequence of length $k+1$ of $1, 11, 111, 1111, ...$).

2. Divide Me

How many numbers in $[360]$ are divisible by:

(a) 4, 6, and 9?

(b) 4, 6, or 9?

(c) Neither 4, 6, nor 9?

3. Keep Drawing Cards...

How many cards must you draw from a standard 52-card deck (4 suits and 13 cards of each suit) until you are guaranteed to have:

(a) A single pair? (e.g., AA, 99, JJ)

(b) Two (different) pairs? (e.g., AAKK, 9933, 44QQ)

(c) A full house (a triple and a pair)? (e.g., AAAKK, 99922, 555JJ)

(d) A straight (5 in a row, with the lowest being A,2,3,4,5 and the highest being 10,J,Q,K,A)?

(e) A flush (5 cards of the same suit)? (e.g., 5 hearts, 5 diamonds)

(f) A straight flush (a straight but all cards of the same suit)?

4. Acing the Exams

In a town of 351 students (the number of students, not ones taking CSE 351), every student aces the midterm, final, or both. If 331 of the students ace the midterm and 45 ace the final, how many people who aced the midterm did not ace the final as well?

5. Friends

Show that in a group of n people (who may be friends with any number of other people), two must have the same number of friends.

6. Senate Committee Assignments

There are 51 senators in a senate. The senate needs to be divided into m committees such that each senator is on exactly one committee. Each senator hates exactly three other senators. (If senator A hates senator B, then senator B does 'not' necessarily hate senator A.) Find the smallest m such that it is always possible to arrange the committees so that no senator hates another senator on his or her committee.

7. Spades and Hearts

Given 3 different spades and 3 different hearts, shuffle them. Compute $\Pr(E)$, where E is the event that the suits of the shuffled cards are in alternating order. What is your sample space?

8. Congressional Tea Party

Twenty politicians are having a tea party, 6 Democrats and 14 Republicans.

(a) If they only give tea to 10 of the 20 people, what is the probability that they only give tea to Republicans?

(b) If they only give tea to 10 of the 20 people, what is the probability that they give tea to 8 Republicans and 2 Democrats?

9. Dinner Party

At a dinner party, the n people present are to be seated uniformly spaced around a circular table. Suppose there is a nametag at each place at the table and suppose that nobody sits down at the correct place. Show that it is possible to rotate the table so that at least two people are sitting in the correct place.