

## CSE 312: Foundations of Computing II

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### QuickCheck: Combinatorics Solutions

#### 0. Apples & Baskets & Pears

Suppose you have 13 (indistinguishable) apples and 5 (distinguishable) baskets. How many ways are there to put all apples into the baskets...

- (a) with at least two apples and at most three in each basket?

**Solution:**

Since each basket has at least two apples, we have three remaining apples to place. Since every basket can also have at most three apples, you can only add one more to each basket. Therefore, we have

$$\binom{5}{3}$$

Now suppose you also have 7 (indistinguishable) pears. How many ways are there to put all apples and all pears into the baskets...

- (b) with at least one apple and one pear in each basket?

**Solution:**

We can start by placing one apple and one pear into each basket, leaving us with 8 apples and 2 pears left. Now, we can treat this as two stars and bars problems.

We start by placing the 8 apples and 4 "bars" that separate the baskets. This leaves us with  $\binom{12}{4}$  options.

For every arrangement of the apples, we can then place the 2 pears (and "4 bars"), which consists of  $\binom{6}{4}$  options. Therefore, the final answer is:

$$\binom{12}{4} \binom{6}{4}$$

- (c) with at most three fruits in each basket?

**Solution:**

Since there can be at most three fruits in a basket, there can be at most 15 total fruits. Since there are 20 total in this scenario, by the pigeonhole principle, this is impossible.