CSE 312

Foundations of Computing II

Hello !! :)

CLT Problems

HW 1 – HW 5 Redos due monday

HW 6 1 week after feedback

HW 7 – HW 8 No redos, but partial credit

We flip a fair coin 40 times. What is the probability that we get between 21 and 25 `HEADS` (inclusive)?

$\sigma_x^2 = np(1-p)$

$\mu_x = np = 20$

We flip a coin with bias 1/3 40 times. What is the probability that we get exactly 20 HEADS?

Bias: $\frac{1}{3}$

$X \sim \text{Binomial}(40, \frac{1}{3})$

$\Pr(X = 20) = \binom{40}{20}\left(\frac{1}{3}\right)^{20}\left(\frac{2}{3}\right)^{20} = 0.0118$

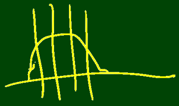
$\Pr(\underset{19.5}{20} \le X \le \underset{\le 20.5}{20}) \approx \Phi(\cdot) - \Phi(\cdot) = 0$

$\Pr(21 \le X \le 25) = \left(\sum \binom{}{}\right)\left(\frac{1}{2}\right)^{40} \approx \boxed{0.396569}$ use binomial

$\Pr_B(21 \le X \le 25) \approx$

use normal

$\Rightarrow \Pr_N\left(\frac{21-20}{\sqrt{10}} \le \frac{X-\mu_x}{\sigma_x} \le \frac{25-20}{\sqrt{10}}\right) =$



$\Phi\left(\frac{25-20}{\sqrt{10}}\right) - \Phi\left(\frac{21-20}{\sqrt{10}}\right) \approx \boxed{0.31897}$

This approximation sucks. To do better, we can abuse the same trick we did when we wrote the code last lecture.

In particular, we can "round" non-integers to integers by treating $[a, \ldots, b]$ as approximately equal to $\left[a - \frac{1}{2}, b + \frac{1}{2}\right]$.

We flip a coin with bias $1/3$ 40 times. What is the probability that we get exactly 20 HEADS?

Bias: $\frac{1}{3}$ exactly 20

exact: 0.0118

$\mu_x = \frac{40}{3}$

$20 \leq x \leq 20$

$\sigma_x = \sqrt{\frac{80}{9}}$

$\Pr(19.5 \leq x \leq 20.5) = \Phi\left(\frac{20.5 - \mu_x}{\sigma_x}\right) - \Phi\left(\frac{19.5 - \mu_x}{\sigma_x}\right)$

Bias: $\frac{1}{2}$ between $\underline{21 \text{ and } 25}$ (inclusive)

$\underline{\text{exact}}$: 0.3969

$\Pr(20.5 \leq x \leq 25.5) =$

$\Phi(-x) = 1 - \Phi(x)$

Roll 10 6-sided dice. Let X = total value of all 10 dice. Win if $X \leq 25$ or $X \geq 45$.

$$\Pr(\text{win}) = 1 - \Pr(25 < X < 45)$$

$$\mu_{\text{uniform}} = \frac{a+b}{2}$$

$$\sigma^2_{\text{uni}} = \frac{(b-a+1)^2 - 1}{12}$$

$$= 1 - \Pr(25.5 \leq X \leq 44.5)$$

μ_X

$$\sigma_X^2 = \operatorname{Var}(X) =$$

Each day your computer crashes with probability 10% independently of every other day. What is the probability of at least 87 crash-free days in the next 100 days?

$X \sim \text{Binomial}(100, 0.9)$

$\mu_X = np = 90$

$\sigma_X^2 = np(1-p) = 9$

$\Pr(87 \leq X \leq 100)$

87
100
∞