CSE 312

Foundations of Computing II

MLEs

$$Pr(x|\theta)$$

$$Pr(x|\lambda)$$

$$\vec{X} = x_0, x_1, x_2, \ldots,$$

$$L(\vec{x}|\theta) = \prod_{x_i} P_x(x_i|\theta)$$

Suppose $X_i \sim \text{Bernoulli}(\theta)$

$$L(\vec{x}|\theta) = \prod_{x_i} p_{\vec{x}}(x_i|\theta) = \theta^{n_1}(1-\theta)^{n_0}$$

$$\log L(\vec{x}|\theta) = n_1 \log\theta + n_0(\log(1-\theta))$$

$$\frac{\partial}{\partial\theta}\log L(\vec{x}|\theta) = \frac{-n_0}{1-\theta} + \frac{n_1}{\theta}$$

Set to 0 $\Longrightarrow$ $\hat{\theta} = \frac{n_1}{n_0+n_1} = \frac{n_1}{n}$

Suppose $X_i \sim N(\theta, 1)$

$$L(\vec{x}|\theta) = \prod_{x_i} \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-\theta)^2}{2}}$$

$$\ln L(\vec{x}|\theta) = \sum_{x_i} -\frac{1}{2}\ln(2\pi) - \frac{(x-\theta)^2}{2}$$

$$\frac{\partial}{\partial\theta} \ln L(\vec{x}|\theta) = \sum_{x_i} (x_i - \theta) = 0$$

$$\hat{\theta} = \frac{\sum x_i}{n} \Leftarrow \sum x_i - \theta n = 0$$