CSE 312

Foundations of Computing II

Review Problems

Count the rectangles of all sizes and at all positions that are formed using segments in a grid with m horizontal lines and n vertical lines.

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A rectangle is uniquely described by 2 distinct vertical lines and 2 distinct horizontal lines on the grid. So we can simply select the 2 vertical lines, and then the 2 horizontal lines and multiply these two quantities together by rule of product.

$$\binom{n}{2}\binom{m}{2}$$

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$$

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$$

The left counts the number of ways to choose a committee of size k and a sub-committee of size j.
The right counts the number of ways to first choose a sub-committee and then choose the remaining members of the committee.

Suppose you are taking a multiple-choice test that has c answer choices for each question. In answering a question on this test, the probability that you know the correct answer is p. If you dont know the answer, you choose one at random. What is the probability that you knew the correct answer to a question, given that you answered it correctly?

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$$\frac{p}{p+(1-p)\frac{1}{c}}$$

Adam is trying to get into his locked office. Unfortunately, he is very tired and cannot remember which of his 5 keys open the door. In each of the following cases, what is the pmf of the number of keys Adam will have to try? What about the expected value?

- Suppose he repeatedly selects one key, tries it on the door, then puts the key back (he may try a given key multiple times).
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- This is just Geometric($1/5$). So, the pmf is $p_X(k) = \frac{1}{5}\left(\frac{4}{5}\right)^{k-1}$ and the expected number of keys is 5.
- Note that $p_X(1) = \frac{1}{5}$, $p_X(2) = \frac{4}{5}\frac{1}{4} = \frac{1}{5}$, etc. Therefore, the expected number of keys is $\frac{5\times 6}{5\times 2} = 3$.

Coins

Alice and Bob have $2n+1$ fair coins. Bob tosses $n+1$ coins and Alice tosses the remaining n coins, all indepdendently. What is the probability that after all coins have been tossed Bob will have gotten more HEADS than Alice?

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Since they have different numbers of coins, it is impossible for them to have exactly the same outcomes. So, Bob either tosses more `HEADS` than Alice or more `TAILS`. Those events are equally likely; so, the probability that Bob gets more `HEADS` is $1/2$.