CSE 312

Foundations of Computing II

Law of Large Numbers & Central Limit Theorem

Suppose $X \sim \text{Binomial}(n, p)$. Then, $\mu = \mathbb{E}[X] = np$.

Definition (Chernoff Bound)

For any $0 < \delta < 1$:

$$\Pr(X > (1+\delta)\mu) \le e^{-\frac{\delta^2 \mu}{3}}$$

$$\Pr(X < (1-\delta)\mu) \le e^{-\frac{\delta^2 \mu}{2}}$$

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Then: $\mathbb{E}\left[\frac{X_1 + \cdots + X_n}{n}\right] = \mu$ and $\mathrm{Var}\left(\frac{X_1 + \cdots + X_n}{n}\right) = \frac{\sigma^2}{n}$

- Weak Law of Large Numbers: For any $\varepsilon > 0$, as $n \to \infty$,

$$\Pr\left(\left|\frac{X_1 + \cdots + X_n}{n} - \mu\right| > \varepsilon\right) \to 0$$

- Strong Law of Large Numbers:

$$\Pr\left(\lim_{n\to\infty}\left(\frac{X_1 + \cdots + X_n}{n}\right) = \mu\right) = 1$$

Consider i.i.d. r.v.'s $X_1, X_2, \ldots, X_n$ with $\mathbb{E}[X_i] < \infty$ and $\mathsf{Var}(X_i) < \infty$. Then, as $n \to \infty$:

Central Limit Theorem

Consider i.i.d. r.v.'s $X_1, X_2, \ldots, X_n$ with $\mathbb{E}[X_i] < \infty$ and $\mathrm{Var}(X_i) < \infty$. Then, as $n \to \infty$:

$$\frac{1}{n}\sum_{i=1}^{n} X_i \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$