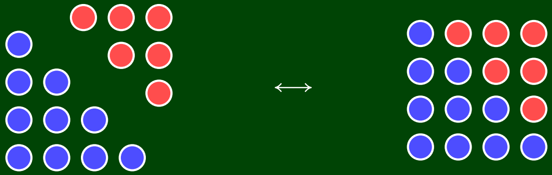
CSE 312

Foundations of Computing II

Counting in Two Ways

(A second time)



Administrivia

- Extra practice section in EEB 037 from 3:30pm - 4:30pm on Thursday. You don't need to sign up; just show up. Yes, the problems will be posted on the website.

- HW Grading

- Adam's Open Door Policy

- HW 1 Out

Outline

Definition (Binomial Theorem)

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Binomial Theorem?

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$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

A Specific Case

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

Binomial Theorem?

Definition (Binomial Theorem)

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

A Specific Case

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

Let's prove this combinatorially!

A Specific Case

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

Proof Sketch.

Set: Binary strings of length n

Partition on the # of 1's

Binomial Theorem?

A Specific Case

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

Proof Sketch.

Set: Binary strings of length n.
Left:

A Specific Case

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

Proof Sketch.

Set: Binary strings of length n.
Left: By definition.
Right:

Binomial Theorem?

A Specific Case

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

Proof Sketch.

Set: Binary strings of length n.
Left: By definition.
Right: Partition on how many bits are 0. □

Even Subsets

$$\sum_{k\geq 0}\binom{n}{2k} = 2^{n-1}$$

Proof Sketch.

Set:

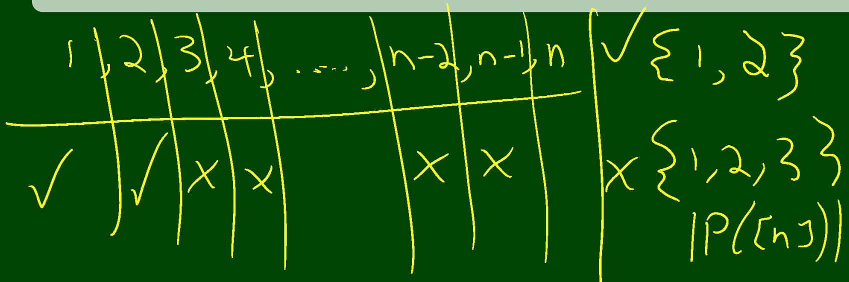
$[n] = \{1, 2, \ldots, n\}$

Even Subsets

$$\sum_{k \geq 0} \binom{n}{2k} = 2^{n-1}$$

Proof Sketch.

Set: Even subsets of $[n]$.
Left:



Even Subsets

Even Subsets

$$\sum_{k \geq 0} \binom{n}{2k} = 2^{n-1}$$

Proof Sketch.

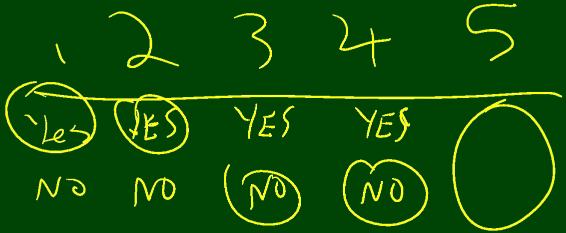
Set: Even subsets of $[n]$.
Left: Partition on how many elements are in the set.
Right:

Even Subsets

$$\sum_{k\geq 0} \binom{n}{2k} = 2^{n-1}$$

Proof Sketch.

Set: Even subsets of $[n]$.
Left: Partition on how many elements are in the set.
Right: Decide whether i is in the set or not up to $n-1$–then, at that point the set we chose is either even or odd. If it's even, we can't include the last element; if it's odd, we must include the last element. □



The Real Deal

Definition (Binomial Theorem)

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

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Proof Sketch.

Set:

4

Definition (Binomial Theorem)

slots

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

# of odd #'s

Proof Sketch.

Set: Consider x odd numbers and y even numbers. We line up n numbers (with replacement!) in a row.

Left:

$$\binom{n}{k} = \binom{n}{n-k}$$

Definition (Binomial Theorem)

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Proof Sketch.

Set: Consider x odd numbers and y even numbers. We line up n numbers (with replacement!) in a row.
Left: For each spot in the row, choose any number.
Right:

The Real Deal

Definition (Binomial Theorem)

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Proof Sketch.

Set: Consider x odd numbers and y even numbers. We line up n numbers (with replacement!) in a row.
Left: For each spot in the row, choose any number.
Right: Partition on how many odd numbers are in the row (k). There are n spots these k numbers could go; choose k of them. Then, determine which k odd numbers are in the row and which $n-k$ even numbers are in the row. □