http://courses.cs.washington.edu/312

Applications:

- inference under uncertainty in AI
  modeled using probability & statistics
  - speech recognition
  - object recognition / vision
  - robot navigation & control
  - any machine learning problem
- simulation
- cryptography
- systems
- big data

....

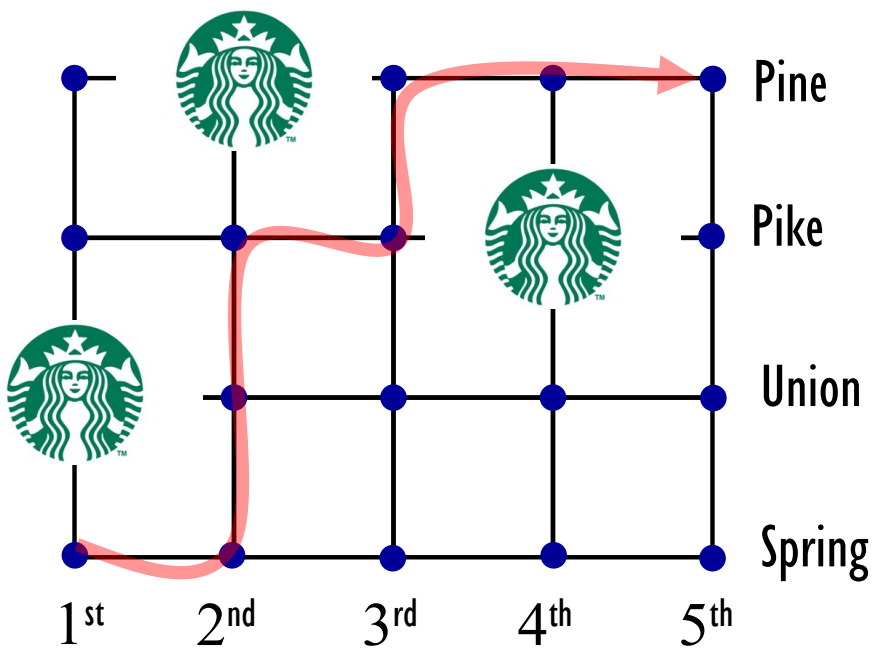
Counting

counting is hard with only 10 fingers

How many ways to do X?

X = "Choose an integer between one and ten."

X = "Walk from 1st and Spring to 5th and Pine."



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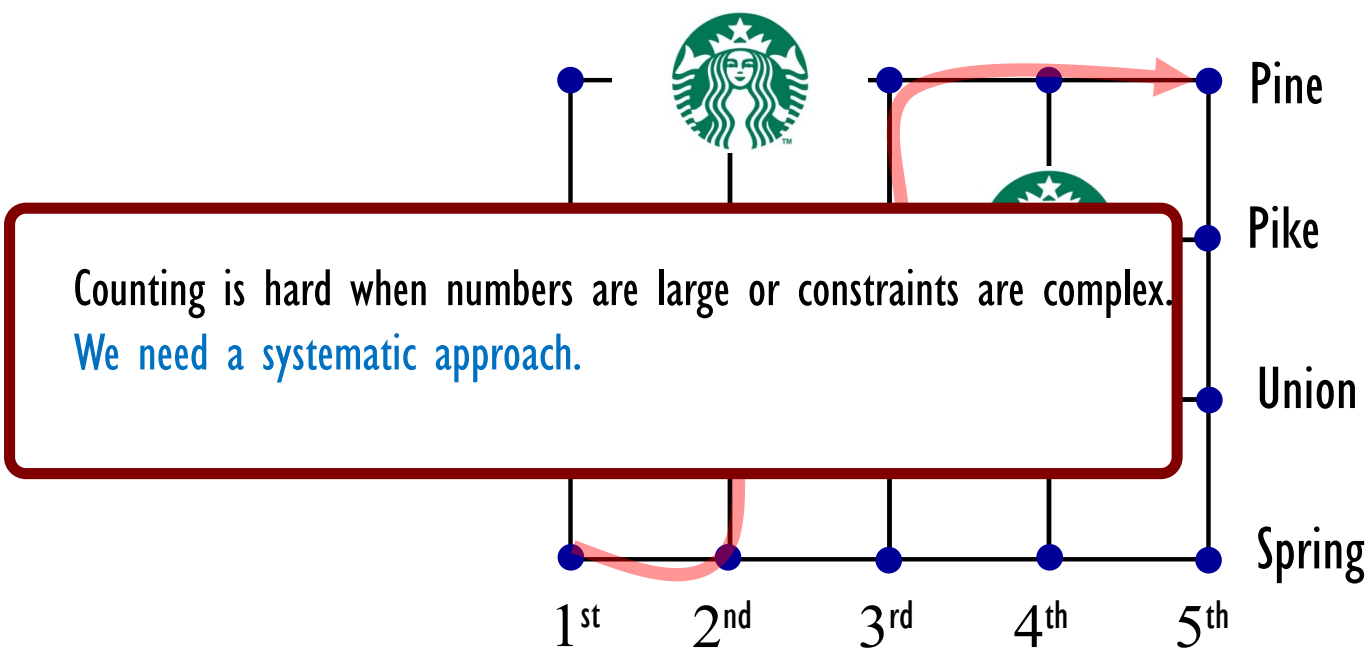


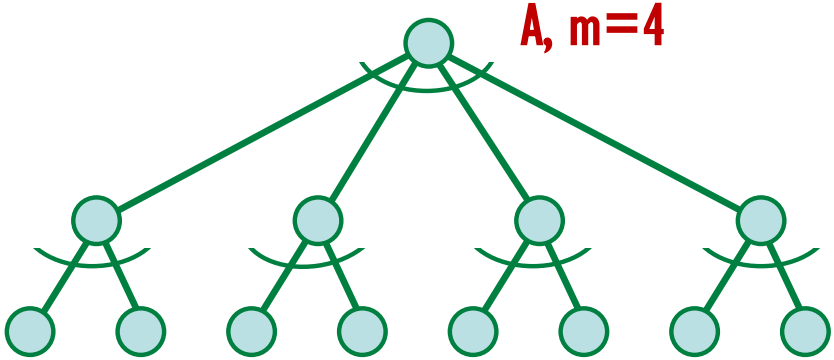
Counting is hard when numbers are large or constraints are complex.
We need a systematic approach.

the basic principle of counting (product rule)

If there are **m** outcomes from some event **A**, followed sequentially by **n** outcomes from some event **B**, then there are...

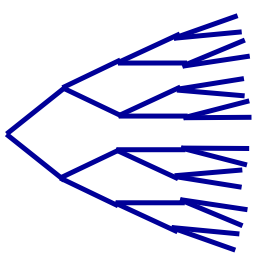
m x n outcomes overall.

5 meats
4 cheeses
2 bread
3 condiments.
$5 \cdot 4 \cdot 2 \cdot 3 = 120$



4 x 2 = 8 outcomes

Generalizes to more events.



How many n-bit numbers are there?

$$2 \cdot 2 \cdot \ldots \cdot 2 = 2^n$$

How many subsets of a set of size n are there?

$\{1, 2, 3, \ldots, n\}$

Set contains **1** or doesn't contain **1**.

Set contains **2** or doesn't contain **2**.

Set contains **3** or doesn't contain **3**…

$$2 \cdot 2 \cdot \ldots \cdot 2 = 2^n$$

How many 4-character passwords are there if each character must be one of
a, b, c, …, z, 0, 1, 2, …, 9 ?

$$36 \cdot 36 \cdot 36 \cdot 36 = 1{,}679{,}616 \approx 1.7 \text{ million}$$

Same question, but now characters cannot be repeated…

$$36 \cdot 35 \cdot 34 \cdot 33 = 1{,}413{,}720 \approx 1.4 \text{ million}$$

How many arrangements of the letters {a,b,c} are possible (using each once, no repeat, order matters)?

a b c	b a c	c a b
a c b	b c a	c b a

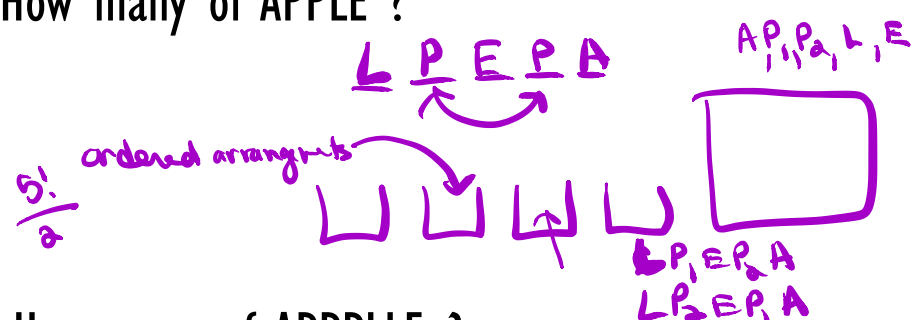
More generally, how many arrangements of n distinct items are possible?

$$n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1 = n! \qquad (n \text{ factorial})$$

Q. How many permutations of PEALS are there?

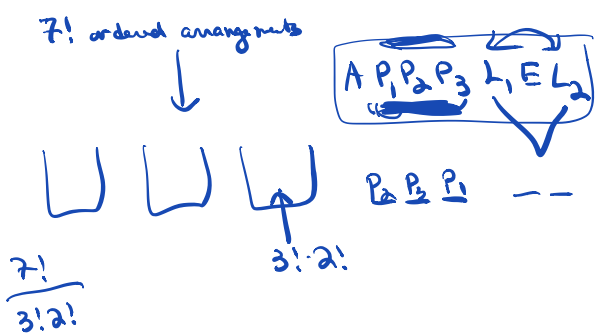
5! _ _ _ _ _

Q. How many of APPLE ?



AP_1P_2LE
AP_2P_1LE

Q. How many of APPPLLE ?



Q. How many permutations of PEALS are there?

$5! = 120$

Q. How many of APPLE ?

$5!/2! = 60$

AP_1P_2LE
AP_2P_1LE

Q. How many of APPPLLE ?

$$\frac{7!}{3!2!1!1!} = 420$$

combinations

Your dark elf avatar can carry three objects chosen from:

How many ways can he/she be equipped?

S F B

SBF
FBS
FSB
...

$\frac{5 \cdot 4 \cdot 3}{3!}$

$\cup \cup \cup$

$3!$

Your dark elf avatar can carry three objects chosen from:

How many ways can he/she be equipped?

$$\frac{5 \cdot 4 \cdot 3}{3!} = \frac{5!}{3! \cdot 2!} = 10$$

Combinations: Number of ways to choose r things from n things

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Pronounced "n choose r" aka "binomial coefficients"

E.g., $\binom{n}{2} = \frac{n(n-1)}{2} = \Theta(n^2)$

Many identities:

$$\binom{n}{r} = \binom{n}{n-r}$$

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

$$\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}$$

$1, 2, \ldots, n$

$n \cdot n-1 \cdots (n-r+1) = \frac{n!}{(n-r)!}$

ordered sequences of r out of n distinct elts.

To get # of unordered sets of r out of n distinct elts divide by r!

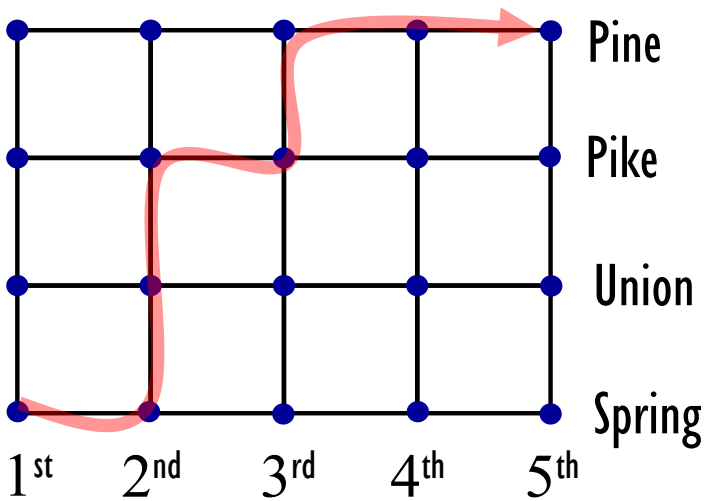
How many ways to walk from 1st and Spring to 5th and Pine only going North and East?

ENNENEE

$\_N_1\_N_2\_N_3\_$

$\frac{7 \cdot 6 \cdot 5}{3!}$

choose positions for north steps.



A: *Changing the visualization often helps.* Instead of tracing paths on the grid above, list choices. You walk 7 blocks; at each intersection choose N or E; must choose N exactly 3 times.

$$\binom{7}{3} = 35$$

counting paths

How many ways to walk from 1st and Spring to 5th and Pine only going North and East, if I want to stop at Starbucks on the way?