More on MLE

$X_1, \dots X_n$ iid. $F(\theta)$

Ex F is $Ber(p)$

$$X_i = \begin{cases} 1 & p \\ 0 & 1-p \end{cases}$$

after the random exp.

see $x_1 = 1,\ x_2 = 0,\ x_3 = 0, \dots, x_n = 1$

these are "samples" from distn

MLE: choice of θ that maximizes $L(x_1, \dots, x_n \mid \theta)$

$$\hat{\theta}(x_1, \dots, x_n) = \frac{\sum_{i=1}^{n} x_i}{n}$$

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$$E[\hat{\theta}(X_1, \dots, X_n)] = \mu$$

all else being equal we prefer unbiased estimators.

$$\hat{\theta}_2(x_1, \dots, x_n) = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

estimator for variance

$\bar{x} = \frac{\sum x_i}{n}$

$$E[\underbrace{\hat{\theta}_2(X_1, \dots, X_n)}]$$

$$\frac{1}{n}\sum_{i=1}^{n} (X_i - \bar{X})^2 = \frac{1}{n}\sum_{i=1}^{n} (X_i^2 - 2X_i\bar{X} + \bar{X}^2)$$

$\frac{1}{n}\sum_{i=1}^{n} X_i$

$$= \frac{1}{n}\sum_{i=1}^{n} X_i^2 - 2\bar{X}\left(\sum_{i=1}^{n} \frac{X_i}{n}\right) + \frac{n\bar{X}^2}{n}$$

$\bar{X}$

$$= \frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2$$

$$Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} Var(\sum X_i) = \frac{1}{n^2} \cdot n \cdot \sigma^2 = \frac{\sigma^2}{n}$$

$$E\left[\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n} \underbrace{E(X_i^2)}_{\sigma^2+\mu^2} - \underbrace{E(\bar{X}^2)}_{\frac{\sigma^2}{n}+\mu^2}$$

$$= \frac{1}{n}\cdot n(\sigma^2+\mu^2) - \frac{\sigma^2}{n} - \mu^2$$

$$= \sigma^2 + \mu^2 - \frac{\sigma^2}{n} - \mu^2$$

$$= \left(1-\frac{1}{n}\right)\sigma^2$$

$\forall$ r.v. Y

$\sigma^2 = \mathrm{Var}(Y) = E(Y^2) - (E(Y))^2$

$E(Y^2) = \sigma^2 + \mu^2$

$\bar{X}$ has mean μ

$\bar{X}$ has variance $\frac{\sigma^2}{n}$

converges to correct value "consistent estimator"

Claim: $\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$ is an unbiased estimator for σ^2