Regarding Problem 4

Let $X$ conditioned on $U = u$ have r.v. density $f_U(-)$.

The expected value of $X$ is given by:

$$E(X) = \int E(X | U = u) f_U(u) du$$

where $E(X | U = u)$ is the conditional expected value of $X$ given $U = u$.

The variance of $X$ is given by:

$$\text{Var}(X) = E((X - E(X))^2)$$

Suppose $\text{Var}(X) = 5$.

The expected value of $X^2$ is given by:

$$E(X^2) = \int E(X^2 | U = u) f_U(u) du$$

The variance of $X^2$ is given by:

$$\text{Var}(X^2) = E(X^2) - (E(X))^2$$

Suppose $E(X^2) = 10$.

The expected value of $X^2$ conditioned on $U = u$ is given by:

$$E(X^2 | U = u) = E((X - 5)^2 | U = u) = E((X - 5)^2)$$
Streaming Algorithms

Distinct Elements

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Y(t) = 0.43 0.43 0.19 0.19 
Y_k(t) = \ldots 
Y(t) 
\hat{\gamma}_t = \frac{1}{Y(t)} - 1

Define \( Y(t) = \min_{0 \leq i < t} h(a_i) \) 

\( E[Y(t)] = \frac{1}{n_{t+1}} \)

\(\gamma(t) = \frac{1}{n_{t+1}} \rightarrow \eta_{t+1} = \frac{1}{Y(t)}\) 

\(\text{Var}(Y(t)) = \frac{1}{(n_{t+1})^2}\)

Consider \( k \) maps \( h_0, h_1, \ldots, h_k \) 

\( Y_i(t) = \min \{ Y_{i}(t) \} \) 

\( E(\bar{Y}(t)) = \frac{1}{n_{t+1}} \)

\( \text{Var}(\bar{Y}(t)) = \frac{1}{k^2} \text{Var}(Y(t)) \) 

\( \text{Cov}(\bar{Y}(t)) = \frac{\text{Cov}(Y(t))}{k^2} \)
\[ \Pr \left( \left| \bar{Y}(t) - \frac{1}{n+1} \right| > k \frac{1}{\sqrt{n(n+1)}} \right) = \frac{1}{4} \]

\[ k = \sqrt{6} \hspace{1cm} \Pr \left( \left| \bar{Y}(t) - \frac{1}{n+1} \right| > \frac{3}{4(n+1)} \right) = \frac{1}{4} \]

\[ \frac{\frac{1}{n+1}}{\frac{1}{2(n+1)}} \leq \bar{Y}(t) \leq \frac{\frac{1}{n+1} + \frac{1}{2(n+1)}}{2} \]

\[ 0 \leq \frac{2}{3(n+1)} \leq \frac{1}{2} \leq \frac{3(n+1)}{3} \leq 2(n+1) \]

\[ \hat{n} = \frac{1}{\bar{Y}(t)} - 1 \]

\[ \frac{\frac{1}{3(n+1)} - 1}{\frac{1}{\frac{1}{3(n+1)} - 1}} \leq 2(n+1) - 1 \]

\[ E \left( g(x) \right) \neq g \left( E(x) \right) \]

\[ E \left( \frac{1}{x} \right) \neq \frac{1}{E(x)} \]
\[ Y = \min (X_1, \ldots, X_n) \]

\[ X_i \sim U[0,1] \]

\[ X_i's \text{ are iid.} \]

\[ E(Y) \]

\[ F_Y(y) = \Pr(Y \leq y) \]

\[ = \Pr(\min(X_1, \ldots, X_n) \leq y) \]

\[ = 1 - \Pr(\min(X_1, \ldots, X_n) > y) \]

\[ = 1 - \Pr(X_1 > y, X_2 > y, \ldots, X_n > y) \]

\[ = 1 - (1 - F_X(y))^n \]

\[ \text{for } U(0,1) \]

\[ f_Y(y) = n(1-y)^{n-1} \]

\[ E(Y) = \int_0^1 y n(1-y)^{n-1} dy \]