Continuous Random Variables

Discrete Random vars

(Ω, P) prob space

$X : \Omega \to \mathbb{R}$

e.g. integer valued $\text{Range}(X) = \{x_1, \dots, x_k\}$

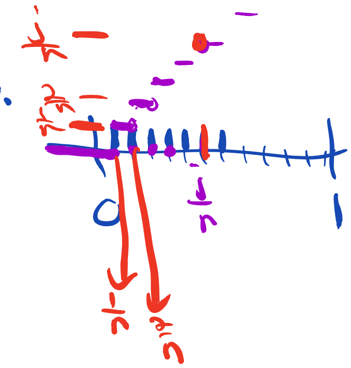
pmf: $p_X(x) = \Pr(X = x)$

$F_X(x) = \sum_{i \mid x_i \le x} p_X(x_i)$

CDF: $F_X(x) = \Pr(X \le x)$

$p_X(x_i) = F_X(x_i) - F_X(x_{i-1})$

Suppose that we want to model cont r.v. that takes values in $[0,1]$



Discrete approx

values r.v. takes $\left\{ \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots, \frac{n}{n} = 1 \right\}$

$$p_X(x) = \begin{cases} \frac{1}{n} & x = \frac{j}{n} \quad 1 \le j \le n \\ 0 & \text{o.w.} \end{cases}$$

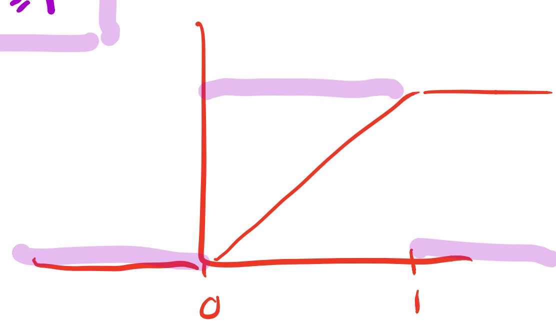
$$F_X(x) = \frac{j}{n} \qquad \frac{j}{n} \le x < \frac{j+1}{n} \qquad 1 \le j \le n$$

$$\boxed{\text{at } x = \frac{j}{n} \quad F_X(x) = x}$$

$$\boxed{\lim_{n \to \infty} F_X(x) = \begin{cases} 0 & x \le 0 \\ x & 0 < x \le 1 \\ 1 & x > 1 \end{cases}}$$

$$\lim_{n \to \infty} p_X(x) = 0$$

$$f_X(x) = \begin{cases} 0 & x < 0 \\ 1 & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$$



p.m.f. doesn't make sense.

instead use $\boxed{\text{probability density fn. } f_X(x)}$

$$f_X(x) = \frac{d}{dx} F_X(x)$$

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

Discrete

$$F_X(x) = \sum_{i \mid x_i \le x} p_X(x_i)$$

$$p_X(x_i) = F_X(x_i) - F_X(x_{i-1})$$

Properties:

$F_X(x)$ is monotone $\nearrow$ from 0 to 1

$f(x) \ge 0$

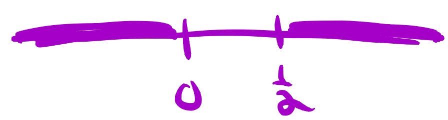
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Pr(a \le X \le b) = \int_a^b f(x)\,dx$$

$$\int_0^{1/2} c\,dx = 1.$$

$$cx\Big|_0^{1/2} = \frac{c}{2} = 1$$

$$f(x) = \begin{cases} 2 & 0 < x \le \frac{1}{2} \\ 0 & \text{o.w.} \end{cases}$$



$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx \qquad E(X) = \sum_{x \in \text{Range}(X)} x\, p_X(x)$$

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f(x)\,dx \qquad E(g(X)) = \sum_{x \in \text{Range}} g(x)\, p_X(x)$$

X cont. r.v. $\qquad f(x) = \begin{cases} c(4x-2x^2) & 0<x<2 \\ 0 & \text{o.w.} \end{cases}$

What is c?

$$2c\int_0^2 (2x-x^2)\,dx = 1$$

$$2c\left[x^2 - \frac{x^3}{3}\right]\Bigg|_0^2 = 2c\left(4-\frac{8}{3}\right) = 8c\left(1-\frac{2}{3}\right)$$

$$= \frac{8c}{3}$$

$$\Rightarrow c = \frac{3}{8}$$

$$\Pr(X \geq 1) = \int_1^2 f(x)\,dx = \int_1^2 \frac{3}{8}(4x-2x^2)\,dx$$

$$E(X) = \int_0^2 x f(x)\,dx = \int_0^2 x\,\frac{3}{8}(4x-2x^2)\,dx$$