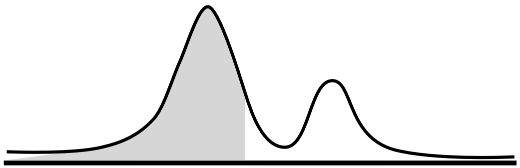
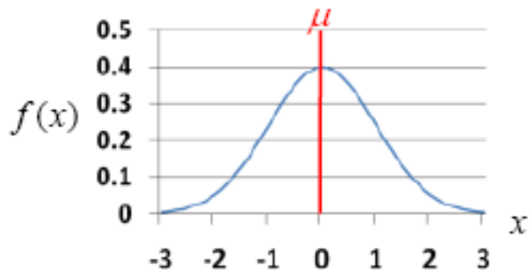
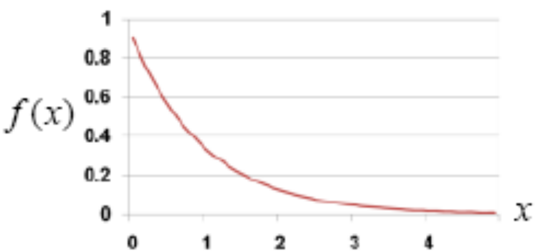
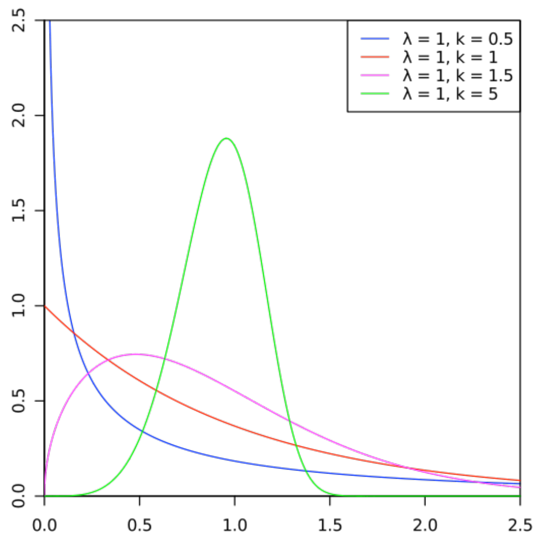





# continuous random variables



*Discrete* random variable: takes values in a finite or countable set, e.g.

$X \in \{1,2, \ldots, 6\}$ with equal probability

X is positive integer i with probability $2^{-i}$

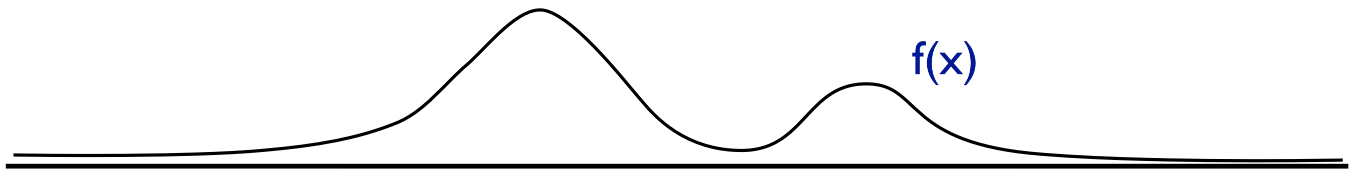
*Continuous* random variable: takes values in an uncountable set, e.g.

X is the weight of a random person (a real number)

X is a randomly selected point inside a unit square

X is the waiting time until the next packet arrives at the server

$f(x): R \rightarrow R$, the *probability density function* (or simply "density")




Require:

$f(x) \geq 0$, and

$\int_{-\infty}^{+\infty} f(x)\, dx = 1$

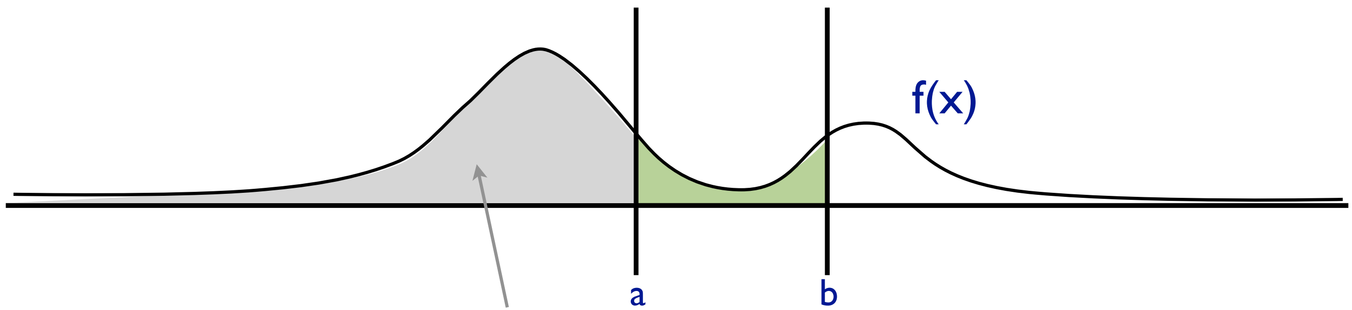
I.e., distribution is:

← nonnegative, and

← normalized,

just like discrete PMF

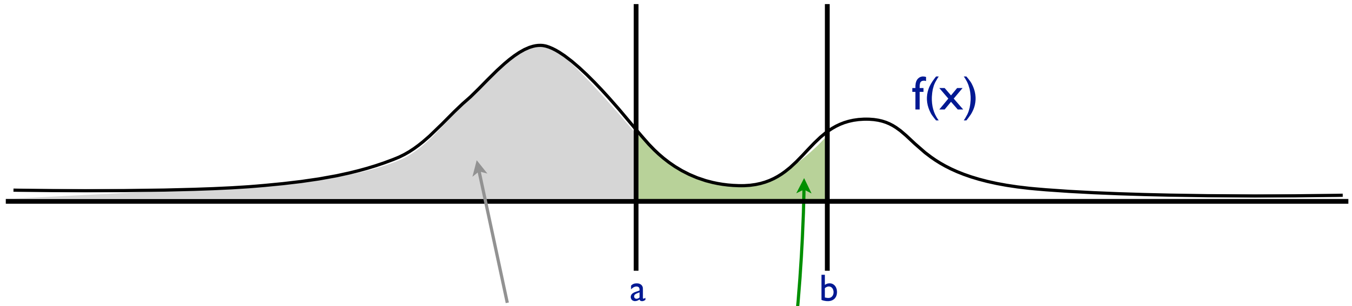
F(x): the *cumulative distribution function* (aka the "distribution")




$F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)\,dx$ (Area left of a)

$P(a < X \leq b) = F(b) - F(a)$

F(x): the *cumulative distribution function* (aka the “distribution”)




$F(a) = P(X \le a) = \int_{-\infty}^{a} f(x)\,dx$ (Area left of a)

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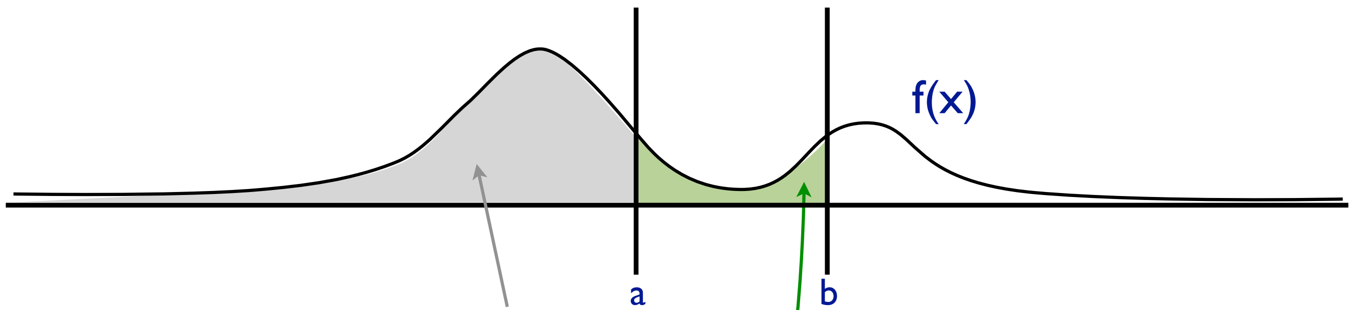


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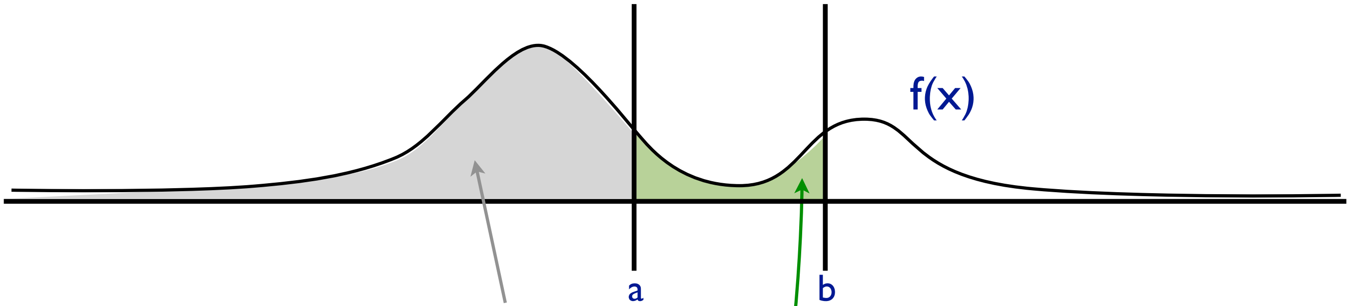
$P(a < X \leq b) = F(b) - F(a)$ (Area between a and b)

Relationship between f(x) and F(x)?

F(x): the *cumulative distribution function* (aka the "distribution")




$F(a) = P(X \le a) = \int_{-\infty}^{a} f(x)\,dx$ (Area left of a)

$P(a < X \le b) = F(b) - F(a)$ (Area between a and b)

A key relationship:

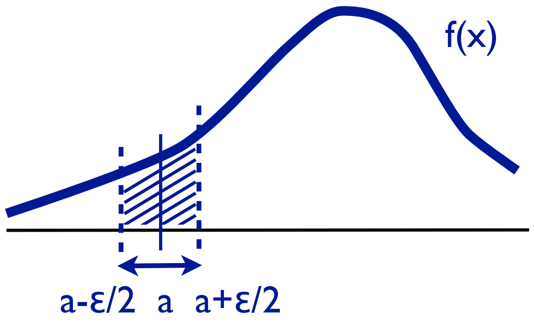
$f(x) = \frac{d}{dx} F(x)$, since $F(a) = \int_{-\infty}^{a} f(x)\,dx$, Densities are *not* probabilities; e.g. may be > 1

$P(X = a) = \lim_{\varepsilon \to 0} P(a-\varepsilon < X \leq a) = F(a)-F(a) = 0$

I.e., the probability that a continuous r.v. falls *at* a specified point is *zero*.

But

the probability that it falls *near* that point is *proportional to the density*:



Densities are *not* probabilities; e.g. may be > 1

$P(X = a) = \lim_{\varepsilon \to 0} P(a-\varepsilon < X \leq a) = F(a)\text{-}F(a) = 0$

I.e.,

the probability that a continuous r.v. falls *at* a specified point is *zero.*

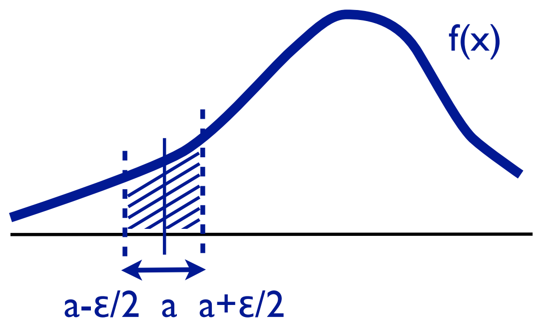
But

the probability that it falls *near* that point is *proportional to the density*:

$P(a - \varepsilon/2 < X \leq a + \varepsilon/2) =$

$F(a + \varepsilon/2) - F(a - \varepsilon/2)$

$\approx \varepsilon \cdot f(a)$




I.e., in a large random sample, expect more samples where density is higher (hence the name "density").

Much of what we did with discrete r.v.s carries over almost unchanged, with $\Sigma_x \ldots$ replaced by $\int \ldots dx$

E.g.

For discrete r.v. X, $E[X] = \Sigma_x \, xp(x)$

For continuous r.v. X, $E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \, dx$

Much of what we did with discrete r.v.s carries over almost unchanged, with $\Sigma_x$... replaced by $\int$... dx

E.g.

For discrete r.v. X, $E[X] = \Sigma_x \, xp(x)$

For continuous r.v. X, $E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

Why?

(a) We define it that way

(b) The probability that X falls "near" x, say within x±dx/2, is ≈f(x)dx, so the "average" X should be ≈ Σ xf(x)dx (summed over grid points spaced dx apart on the real line) and the limit of that as dx→0 is ∫xf(x)dx

Continuous random variable X has density f(x), and

$$\Pr(a \le X \le b) = \int_a^b f(x)\, dx$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$$

$$\sum_x x^2 p_X(x)$$

## Linearity

$$E[aX+b] = aE[X]+b$$

$$E[X+Y] = E[X]+E[Y]$$

still true, just as for discrete

## Functions of a random variable

$$E[g(X)] = \int g(x)f(x)dx$$

just as for discrete, but w/integral

Alternatively, let $Y = g(X)$, find the density of $Y$, say $f_Y$, and directly compute $E[Y] = \int y f_Y(y)dy$.

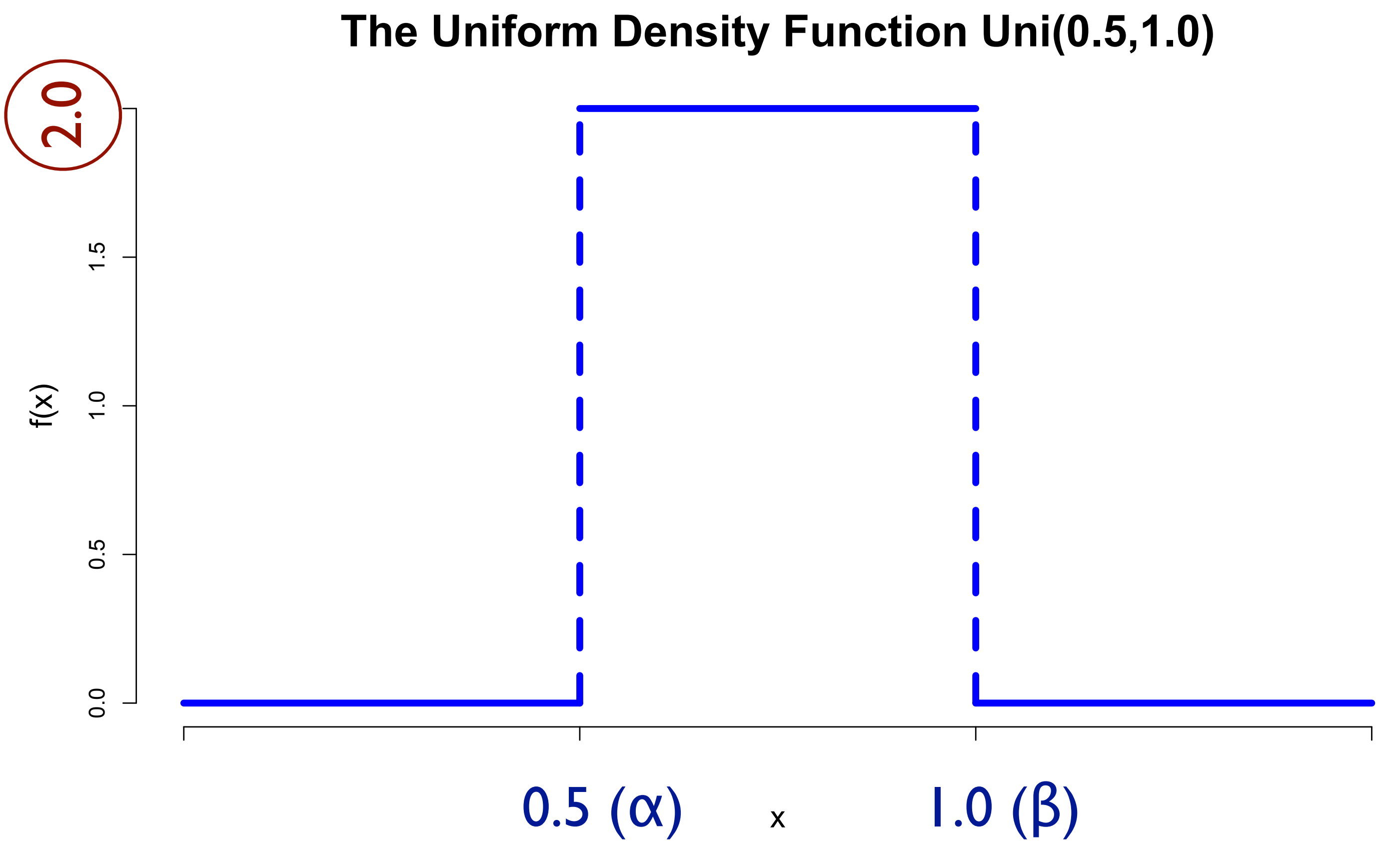
Definition is same as in the discrete case

$Var[X] = E[(X-\mu)^2]$ where $\mu = E[X]$

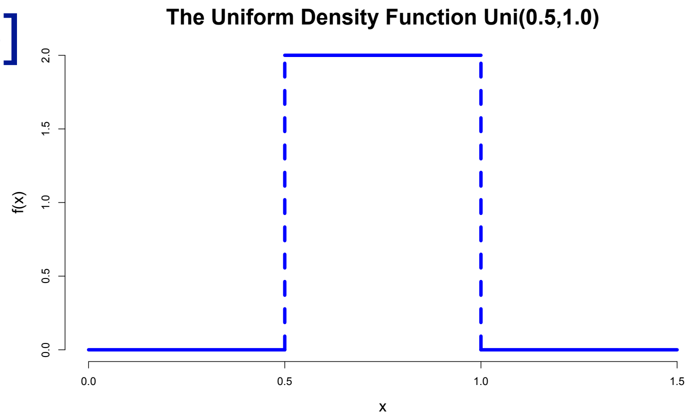
Identity still holds:

$Var[X] = E[X^2] - (E[X])^2$ proof "same"

X ~ Uni(α,β) is uniform in [α,β] $f(x) = \begin{cases} \frac{1}{\beta-\alpha} & x \in [\alpha, \beta] \\ 0 & \text{otherwise} \end{cases}$



X ~ Uni(α,β) is uniform in [α,β]




$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & x \in [\alpha, \beta] \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(a \le X \le b) = \int_a^b f(x)\,dx = \frac{b-a}{\beta-\alpha}$$

if $\alpha \le a \le b \le \beta$:

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \frac{\alpha+\beta}{2}$$

Yes, you should review your basic calculus; e.g., these 2 integrals would be good practice.

Radioactive decay: How long until the next alpha particle?

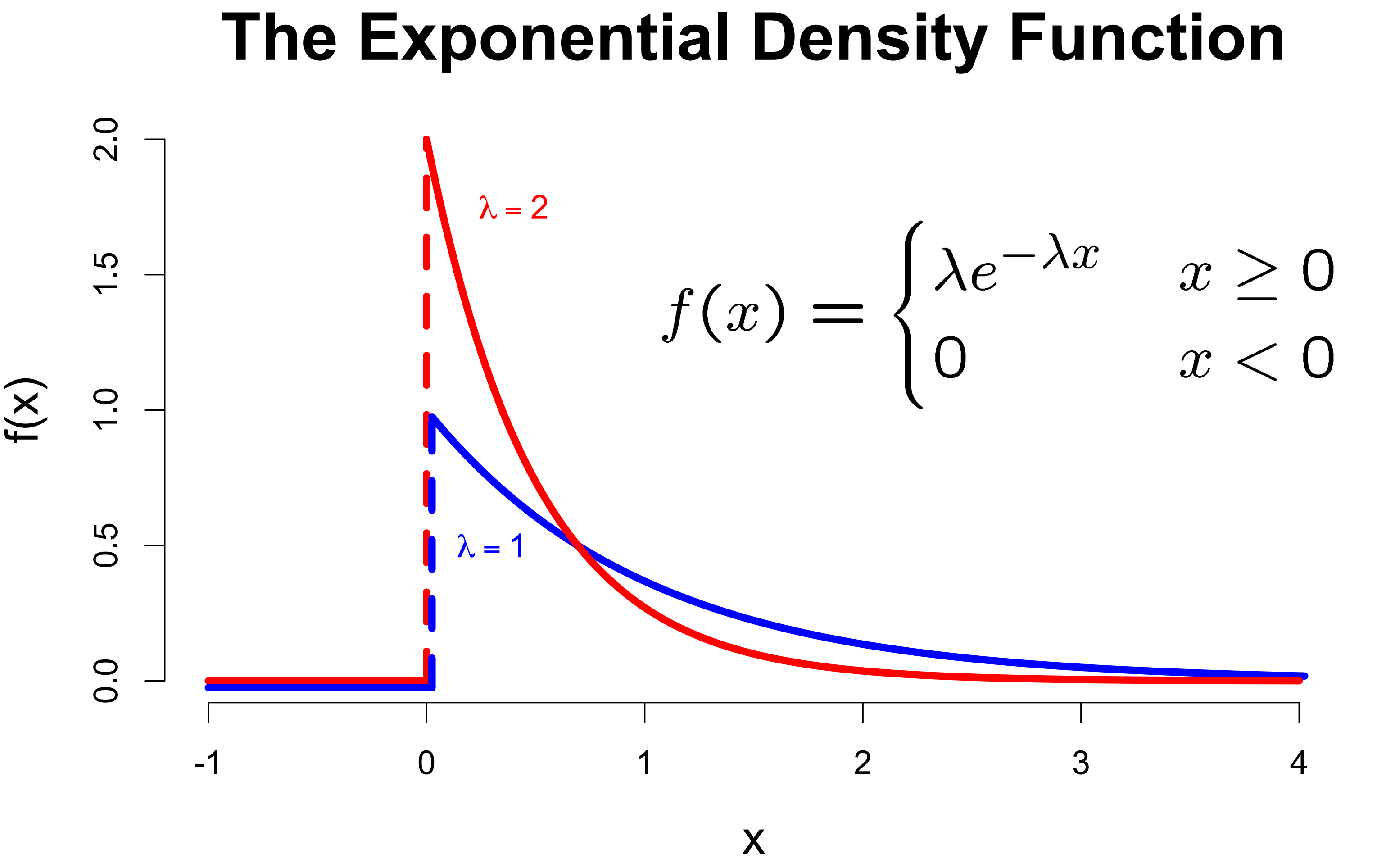
Customers: how long until the next customer/packet arrives at the checkout stand/server?

Buses: How long until the next #71 bus arrives on the Ave?

Yes, they have a schedule, but given the vagaries of traffic, riders with-bikes-and-baby-carriages, etc., can they stick to it?

Assuming events are independent, happening at some fixed *average* rate of λ per unit time – the waiting time until the next event is exponentially distributed (next slide)

X ~ Exp(λ)


The Exponential Density Function
λ = 2
λ = 1
$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$
f(x)
x
0.0
0.5
1.0
1.5
2.0
-1
0
1
2
3
4

X ~ Exp(λ)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$E[X] = \frac{1}{\lambda} \qquad \mathrm{Var}[X] = \frac{1}{\lambda^2}$$

$$\Pr(X \geq t) = e^{-\lambda t} = 1\text{-}F(t)$$

Memorylessness:

$$\Pr(X > s + t \mid X > s) = \Pr(X > t)$$

*Assuming* exp distr, if you've waited *s* minutes, prob of waiting *t* more is exactly same as *s = 0*

$Y \sim Geo(p)$

$\Pr(Y > k) = (1-p)^k$

$E(Y) = \frac{1}{p}$

Same process, different measures:

Poisson: *how many* events in a *fixed time*;

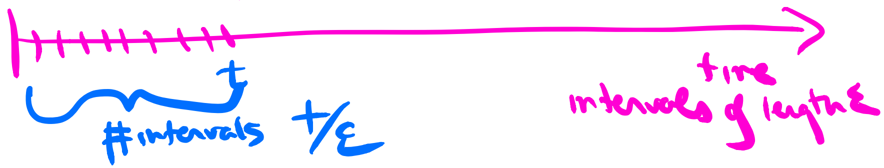
Exponential: *how long* until the *next event*

λ is avg # per unit time;

1/λ is mean wait

$X \sim Exp(\lambda)$

- in tiny interval of length $\varepsilon$ $\Pr(\text{event}) = \lambda\varepsilon$
- in disjoint intervals, prob events indep




$$\Pr(X > t) = (1-\lambda\varepsilon)^{t/\varepsilon}$$

$1 - x \approx e^{-x}$

lim as $\varepsilon \to 0$

$$= \left(e^{-\lambda\varepsilon}\right)^{t/\varepsilon} = e^{-\lambda t}$$

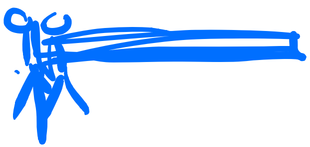
$$F_X(t) = \Pr(X \le t) = 1 - e^{-\lambda t}$$

$$f_X(t) = \frac{d}{dt} F_X(t) = \lambda e^{-\lambda t}$$

$$E(X) = \underbrace{\frac{1}{\lambda\varepsilon}}_{\text{Exp \# trials}} \cdot \underbrace{\varepsilon}_{\text{time per trial}} = \frac{1}{\lambda}$$

Time it takes to check someone out at a grocery store is exponential with ~~with~~ an average value of 10. minutes

If someone arrives to the line immediately before you, what is the probability that you will have to wait between 10 and 20 minutes?



$$T \sim \mathrm{Exp}\left(\frac{1}{10}\right)$$

$$\Pr(10 \le T \le 20) = \int_{10}^{20} \frac{1}{10} e^{-x/10}\, dx$$

Time it takes to check someone out at a grocery store is exponential with with an average value of 10.

If someone arrives to the line immediately before you, what is the probability that you will have to wait between 10 and 20 minutes?

$$T \sim exp(10^{-1})$$

$$Pr(10 \leq T \leq 20) = \int_{10}^{20} \frac{1}{10} e^{-\frac{x}{10}} dx$$

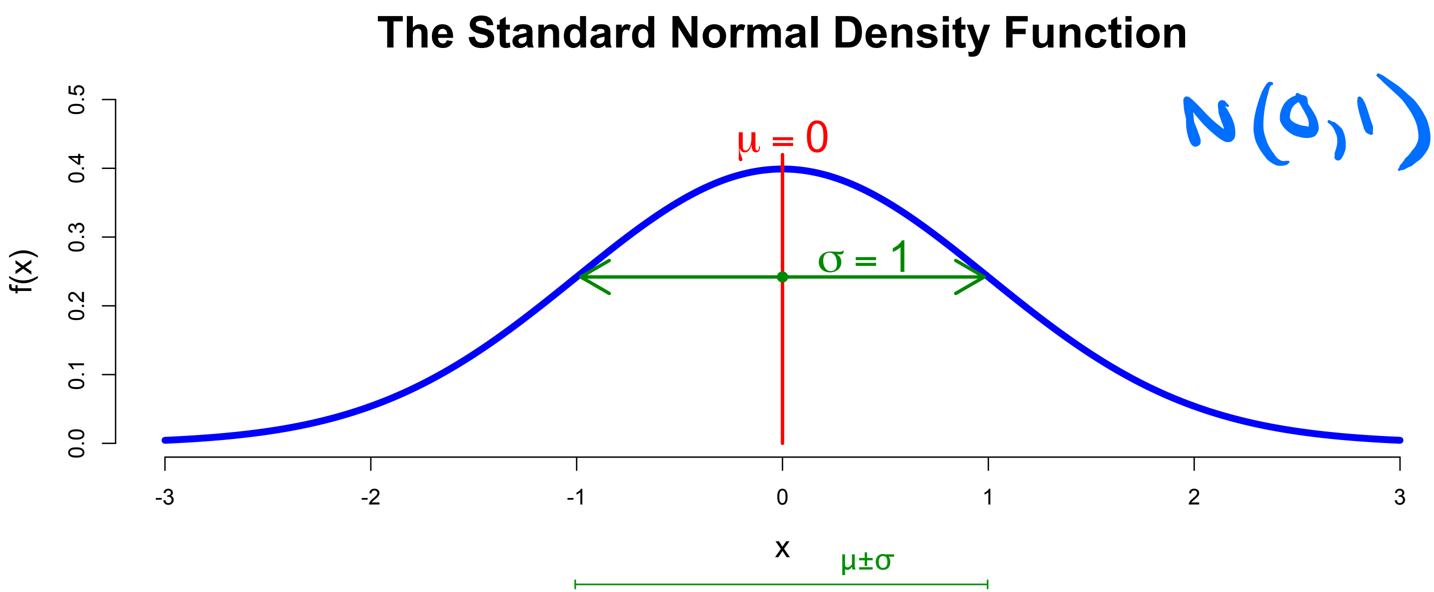
$$y = \frac{x}{10} \qquad\qquad dy = \frac{1}{10} dx$$

$$Pr(10 \leq T \leq 20) = \int_{1}^{2} e^{-y} dy = -e^{-y}\Big|_{1}^{2} \quad = (e^{-1} - e^{-2})$$

X is a normal (aka Gaussian) random variable $\boxed{X \sim N(\mu, \sigma^2)}$

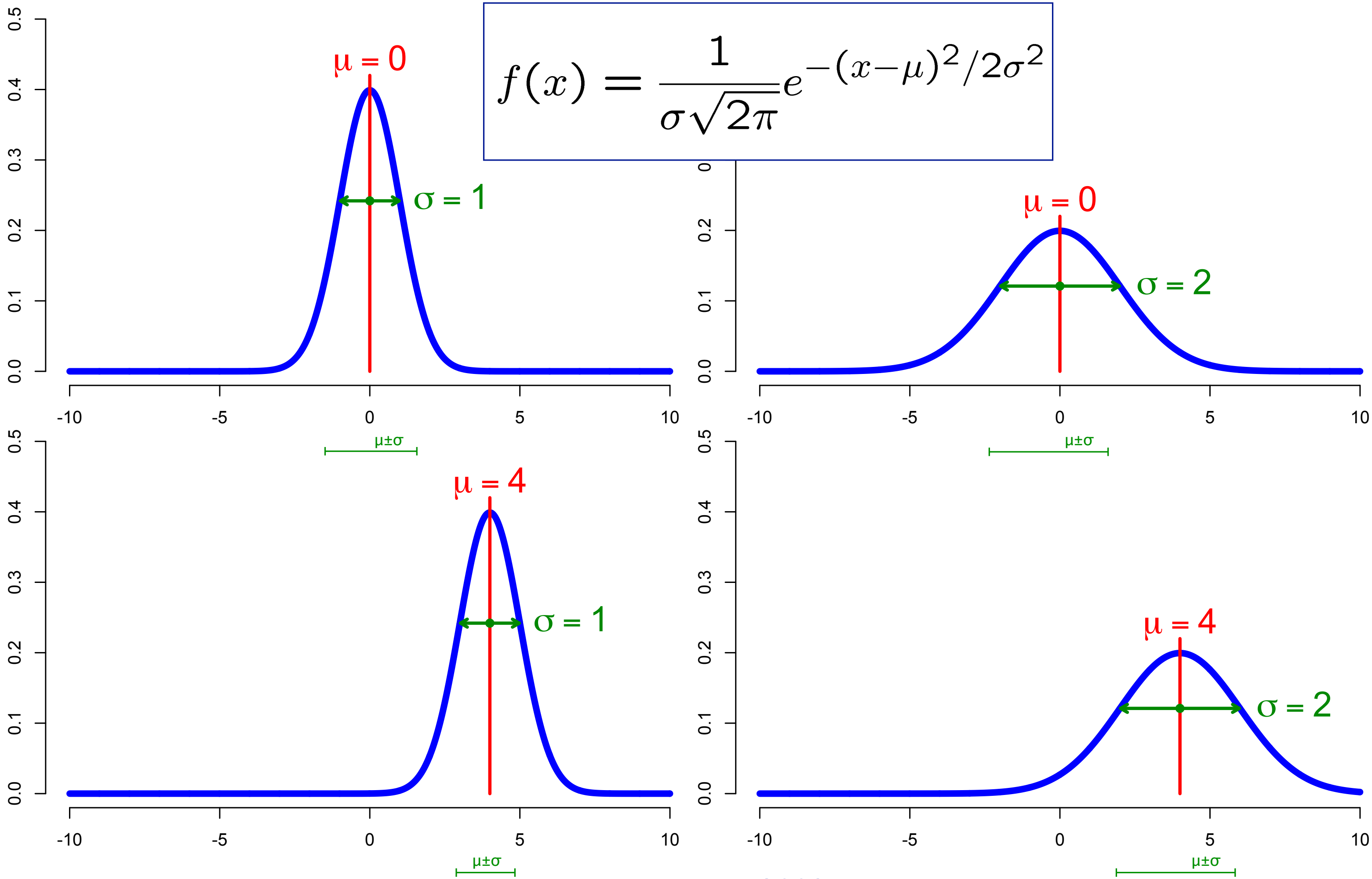
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$E[X] = \mu \qquad \mathrm{Var}[X] = \sigma^2$$



# changing μ, σ

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$




density at μ is ≈ .399/σ

X is a normal random variable $X \sim N(\mu,\sigma^2)$

mean variance

$Y = aX + b$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$E[Y] = a\mu + b$

$Var[Y] = a^2\sigma^2$

X is a normal random variable $X \sim N(\mu,\sigma^2)$

$Y = aX + b$

$E[Y] = E[aX+b] = a\mu + b$

$Var[Y] = Var[aX+b] = a^2\sigma^2$

$Y \sim N(a\mu + b, a^2\sigma^2)$

E[ ], Var[ ] as expected;
"normality" is the surprise

$\boxed{\text{Important special case: } Z = (X-\mu)/\sigma \sim N(0,1)}$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

X is a normal random variable $X \sim N(\mu,\sigma^2)$

$Y = aX + b$

$E[Y] = E[aX+b] = a\mu + b$

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E[ ],Var[ ] as expected;
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Important special case: $Z = (X-\mu)/\sigma \sim N(0,1)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$Z \sim N(0,1)$ "*standard* (or *unit*) normal"

Use $\Phi(z)$ to denote CDF, i.e.

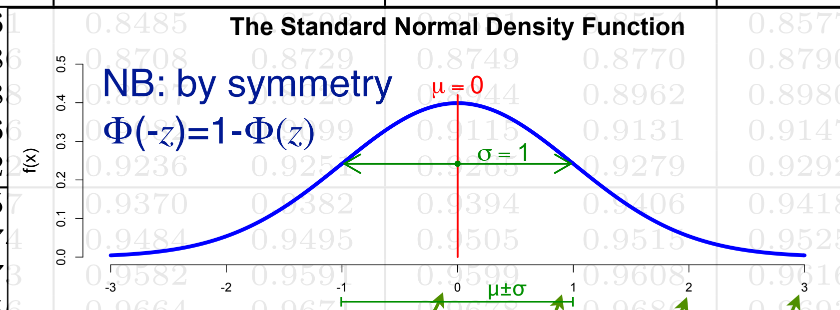
$$\Phi(z) = \Pr(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

no closed form ☹

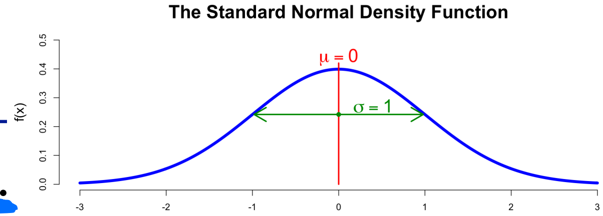
# Table of the Standard Normal Cumulative Distribution Function $\Phi(z)$

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | [illegible] | 0.7517 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | [illegible] | [illegible] | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | [illegible] | [illegible] | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 |

$\Phi(.46)$




E.g., see B&T p155, p531




X normal with mean 3 and variance 9.
What is

Pr (X > 0) $= Pr\left(\boxed{\frac{X-3}{3}} > \frac{0-3}{3}\right) = Pr(Z > -1)$

$Z \sim N(0,1)$

$= Pr(Z < 1) = 0.8413$

Pr (2 < X < 5) $= Pr\left(\frac{2-3}{3} < \boxed{\frac{X-3}{3}} < \frac{5-3}{3}\right) = Pr\left(-\frac{1}{3} < Z < \frac{2}{3}\right)$

$= \Phi\left(\frac{2}{3}\right) - \underbrace{\Phi\left(-\frac{1}{3}\right)} = 0.745 - \underbrace{\left(1 - \Phi\left(\frac{1}{3}\right)\right)}_{0.6664}$

Pr ( |X-3| > 6) $= Pr(X > 9 \text{ or } X < -3)$

$= Pr(X > 9) + Pr(X < -3)$

# Table of the Standard Normal Cumulative Distribution Function $\Phi(z)$

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| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 |

X normal with mean 3 and variance 9.

$$Pr(2 < X < 5) = Pr(\frac{2-3}{3} < \frac{X-3}{3} < \frac{5-3}{3}) = Pr(-\frac{1}{3} < Z < \frac{2}{3})$$

$$= \Phi\left(\frac{2}{3}\right) - \Phi\left(-\frac{1}{3}\right) \quad = \Phi\left(\frac{2}{3}\right) - \left(1 - \Phi\left(\frac{1}{3}\right)\right)$$

$$= 0.7456 - (1 - 0.6293)$$

$$Pr(X > 0) = Pr(Z > \frac{0-3}{3}) = Pr(Z > -1) = Pr(Z < 1) = 0.8413$$

$$Pr(|X-3| > 6) = Pr(X > 9) + Pr(X < -3) =$$
$$Pr(Z > \frac{9-3}{3}) + Pr(Z < \frac{-3-3}{3})$$

X normal with mean 5 and variance $\sigma^2$

If $\Pr(X > 9) = 0.2$, then approximately what is $\sigma^2$ ?

$X \sim N(5, \sigma^2)$

$\Pr(X > 9) = 0.2$

$\Pr\left( \frac{X-5}{\sigma} > \frac{9-5}{\sigma} \right) = 1 - \Phi\left(\frac{9-5}{\sigma}\right) = 0.2$

$Z > \frac{9-5}{\sigma}$

$\Phi\left(\frac{9-5}{\sigma}\right) = 0.8$

$\Phi(0.85) \approx 0.8$

$\frac{9-5}{\sigma} = 0.85$

X normal with mean 5 and variance $\sigma^2$
If Pr(X> 9) = 0.2, then approximately what is $\sigma^2$ ?

$$Pr(X > 9) = Pr(\frac{X-5}{\sigma} > \frac{9-5}{\sigma}) = 0.2$$

$$1 - \Phi\left(\frac{9-5}{\sigma}\right) = 0.2$$

$$\Phi\left(\frac{9-5}{\sigma}\right) = 0.8$$

Look up in N(0,1) table to find out what v gives $\Phi(v) = 0.8$

Set $\frac{9-5}{\sigma} = v$ and solve for $\sigma$

pdf vs cdf

$f(x) = \frac{d}{dx} F(x)$ $\qquad$ $F(a) = \int_{-\infty}^{a} f(x)\, dx$

sums become integrals, e.g.

$E[X] = \Sigma_x\, xp(x)$ $\qquad$ $E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

most familiar properties still hold, e.g.

$E[aX+bY+c] = aE[X]+bE[Y]+c$

$Var[X] = E[X^2] - (E[X])^2$

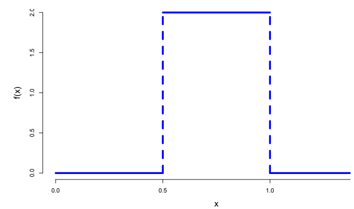
# Three important examples

## X ~ Uni(α,β) uniform in [α,β]

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & x \in [\alpha, \beta] \\ 0 & \text{otherwise} \end{cases}$$

$E[X] = (\alpha+\beta)/2$

$Var[X] = (\alpha-\beta)^2/12$

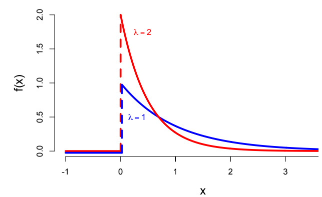



## X ~ Exp(λ) exponential

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$E[X] = \frac{1}{\lambda}$$

$$\mathrm{Var}[X] = \frac{1}{\lambda^2}$$




## X ~ N(μ, σ²) normal (aka Gaussian, aka the big Kahuna)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$E[X] = \mu$$

$$\mathrm{Var}[X] = \sigma^2$$