Markov Inequality:

When to use it: If your variable is **non-negative** and **you know its mean but can't use its variance** (or you don't want to use variance), you can say that it is **unlikely to take on a value that is many times larger than its mean** (be larger than $aE[X]$ where a is big).

For nonnegative random variable X and any $a > 0$,

$$\mathbf{Pr}[X \geq a] \leq \frac{E[X]}{a}.$$

Equivalently, for any $b > 0$,

$$\mathbf{Pr}[X \geq bE[X]] \leq \frac{1}{b}.$$

Chebyshev's Inequality:

When to use it: If **you know your variable's mean and variance**, you can say that it is **unlikely to deviate too far from its mean in either direction** (have a large value of $|X - E[X]|$), and this likelihood goes up with variance.

For any random variable X and any $a > 0$,

$$\mathbf{Pr}[|X - E[X]| \geq a] \leq \frac{Var[X]}{a^2}.$$

Equivalently, for any $b > 0$,

$$\mathbf{Pr}[|X - E[X]| \geq bVar[X]] \leq \frac{1}{b^2 Var[X]}.$$

Notice that if $Var[X] = \sigma^2$ this becomes

$$\mathbf{Pr}[|X - E[X]| \geq b\sigma] \leq \frac{1}{b^2}.$$

Chernoff Bound:

Chernoff-type bounds take many forms. One example is:

When to use it: If your **variable is distributed binomially, meaning it is the sum of independent Bernoullis** (so $X = \sum X_i$), you can say that it is very **unlikely that it is far from its mean in either direction**.

For any random variable $X \sim \text{Bin}(n, p)$ and any δ between 0 and 1,

$$\mathbf{Pr}[X > (1+\delta)E[X]] \leq e^{-\frac{\delta^2 E[X]}{3}}$$

$$\mathbf{Pr}[X < (1-\delta)E[X]] \leq e^{-\frac{\delta^2 E[X]}{2}}.$$