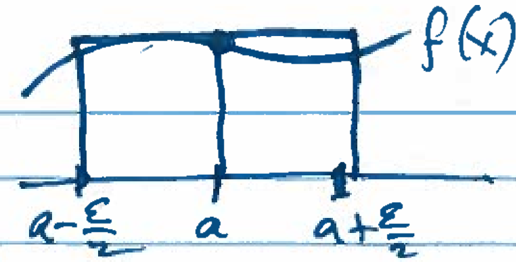


$P(a-\frac{\varepsilon}{2} < X \le a+\frac{\varepsilon}{2})$
$= F(a+\frac{\varepsilon}{2}) - F(a-\frac{\varepsilon}{2})$
$= \int_{a-\frac{\varepsilon}{2}}^{a+\frac{\varepsilon}{2}} f(x)\,dx \approx \varepsilon f(a)$

The probability that X is "near" a is proportional ~~to~~ to $f(a)$.

For continuous r.v.s, usually substitute $\int$ for Σ.

Continuous

$E[X] = \int_{-\infty}^{+\infty} x f(x)\,dx$

$E[aX+b] = aE[X]+b$

$E[X+Y] = E[X]+E[Y]$

$E[g(X)] = \int_{-\infty}^{+\infty} g(x) f(x)\,dx$

$Var(X) = E[(X-\mu)^2] = E[X^2] - (E[X])^2$

Recall discrete

$E[X] = \sum_x x p(x)$

$E[g(X)] = \sum_x g(x) p(x)$

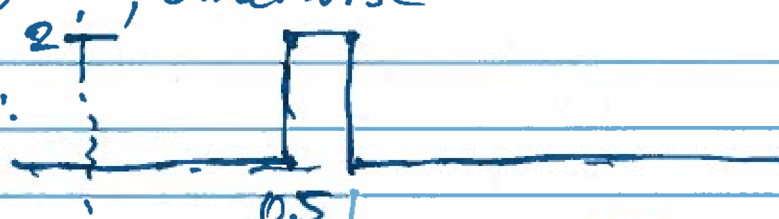
Special continuous distributions:

Uniform

$X \sim Uni(\alpha,\beta)$ has density

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & \text{if } \alpha \le x \le \beta \\ 0, & \text{otherwise} \end{cases}$$

$Uni(0.5,1)$:



$$E[X] = \int_{-\infty}^{+\infty} x f(x)\,dx = \cancel{\int_{0.5}^{1} x\cdot 2\,dx = 2\int_{0.5}^{1} x\,dx} = \int_{\alpha}^{\beta} x\cdot\frac{1}{\beta-\alpha}\,dx = \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta} x\,dx$$

$$= \frac{1}{2(\beta-\alpha)} x^2\Big|_{\alpha}^{\beta} = \frac{\beta^2-\alpha^2}{2(\beta-\alpha)} = \frac{1}{2}(\alpha+\beta)$$

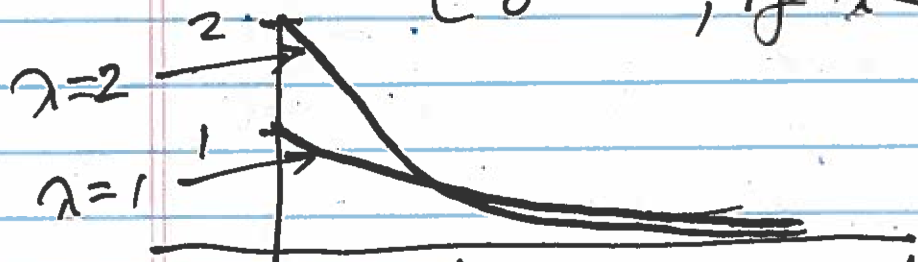
Exponential distribution

Radioactive decay: time until emission of next α particle.

Time until arrival of next packet.

$X \sim Exp(\lambda)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$



$E[X] = \frac{1}{\lambda}$, $Var(X) = \frac{1}{\lambda^2}$

$P(X \geq t) = e^{-\lambda t} = 1 - F(t)$

$F(t) = 1 - e^{-\lambda t} \Rightarrow \frac{d}{dx} F(x) = \frac{d}{dx}(1 - e^{-\lambda x})$
$= \lambda e^{-\lambda x}$

~~Time~~ Probability you have to wait time t for next event is $e^{-\lambda t}$

Memorylessness: If $X \sim Exp(\lambda)$, then
$P(X > s+t \mid X > s) = P(X > t)$.

Connection to Poisson

Exponential is the continuous analog of geometric distribution.