Bayes' Theorem (Rev. Thomas Bayes, c.1701–1761):

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}$$

Proof: $P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{P(E|F)P(F)}{P(E)}$

Corollary:

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|\bar{F})P(\bar{F})}$$

Proof: LTP

Why it's useful: reverses conditioning

Ex: F = disease, E = test result

You might know $P(E|F)$

Want to know $P(F|E)$ if you had a + test result.

Ex: 60% of email is spam

90% of spam has a forged header

20% of nonspam has a forged header.

Let F = forged header, J = spam

What is $P(J|F)$?

$$P(J|F) = \frac{P(F|J)P(J)}{P(F|J)P(J) + P(F|\bar{J})P(\bar{J})}$$

$$= \frac{0.9 \times 0.6}{0.9 \times 0.6 + 0.2(1-0.6)} \approx 0.871$$

prior probability of spam = 0.6

posterior probability = 0.871

genotype
allele = "variant"

Ex: Paternity testing

Child has (A,a) pair for some gene (event B_{Aa})

Mother has (A,A)

2 possible fathers, $F_1=(a,a)$, $F_2=(A,a)$.

$P(F_1)=p$, $P(F_2)=1-p$

$$P(F_1 \mid B_{Aa}) = \frac{P(B_{Aa} \mid F_1)P(F_1)}{P(B_{Aa} \mid F_1)P(F_1)+P(B_{Aa} \mid F_2)P(F_2)}$$

$$= \frac{1\cdot p}{1\cdot p+\frac{1}{2}(1-p)} = \frac{2p}{2p+1-p} = \boxed{\frac{2p}{1+p}}$$ posterior

$$\geq \frac{2p}{1+1} = \boxed{p}$$ prior

For example, if $p=\frac{1}{2}$, $\frac{2p}{1+p}=\frac{2}{3}$