Complementing.

Ex: Assume ♣J is the face-up trump. How many starting Schnapsen hands contain ≥ 1 trump?

$$12{,}240 = \binom{4}{1}\binom{18}{4}$$

18 ← cards remaining in the deck

4 ← cards I still need to be dealt.

♣A ♣K ♡KQJ

♣K ♣A ♡KQJ   Overcounted!

~~♣A ♡K~~

♣A ♣K ♣Q ♡QJ

♣K ♣A ♣Q ♡QJ

♣Q ♣A ♣K ♡QJ

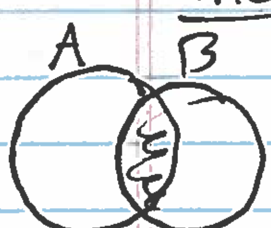
⋮

How many contain 0 trumps? $\binom{15}{5}$

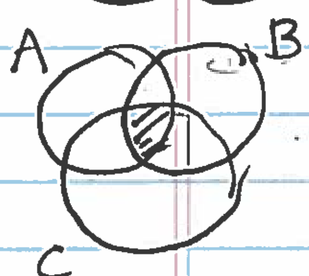
How many contain ≥ 1? $\binom{19}{5} - \binom{15}{5}$

$$= \frac{19\cdot18\cdot17\cdot16\cdot15}{5\cdot4\cdot3\cdot2\cdot1} - \frac{15\cdot14\cdot13\cdot12\cdot11}{5\cdot4\cdot3\cdot2\cdot1} = 8625$$

Inclusion-Exclusion:



$$|A\cup B| = |A| + |B| - |A\cap B|$$



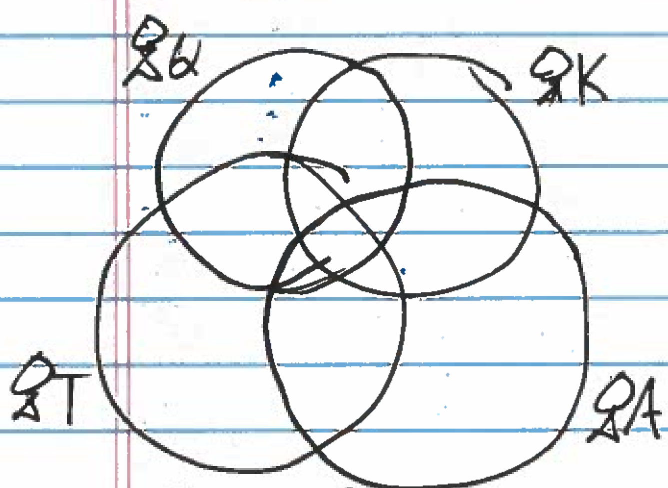
$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$$

In general: + single − pairs + triples − quad + ...

Ex: Correct the overcounting in $\binom{4}{1}\binom{18}{4}$:

$$\binom{4}{1}\binom{18}{4} - \binom{4}{2}\binom{17}{3} + \binom{4}{3}\binom{16}{2} - \binom{4}{4}\binom{15}{1}$$

$$= 12{,}240 - 4080 + 480 - 15 = 8625$$



Pigeonhole Principle: If you have n pigeons and k pigeonholes, where $k<n$, then some pigeonhole has >1 pigeon.

Ex:

5x5 chessboard, 1 flea on each square. When I ring a bell, every flea jumps to an adjacent (horizontally or vertically) square. After that, some square has ≥ 2 fleas.

13 white, 12 black.

13 fleas jump onto 12 squares.