CSE 312: Foundations of Computing II
Quiz Section #4: Discrete Random Variables

<u>Review/Mini-Lecture/Main Theorems and Concepts From Lecture</u>

Random Variable (rv): A numeric function $X: \Omega \to \mathbb{R}$ of the outcome.

Range/Support: The support/range of a random variable X, denoted Ω_X, is the set of all possible values that X can take on.

Discrete Random Variable (drv): A random variable taking on a _______________ (either finite or countably infinite) number of possible values.

Probability Mass Function (pmf) for a discrete random variable X: a function $p_X: \Omega_X \to [0,1]$ with $p_X(x) = P(X = x)$ that maps possible values of a discrete random variable to the probability of that value happening, such that $\sum_x p_X(x) =$ ____.

Expectation (expected value, mean, or average): The expectation of a discrete random variable is defined to be

$$E[X] = \sum_{\_\_} \underline{\hspace{3cm}}$$

The expectation of a function of a discrete random variable $g(X)$ is

$$E[g(X)] = \sum_{\_\_} \underline{\hspace{4cm}}$$

Linearity of Expectation: Let X and Y be random variables, and $a, b, c \in \mathbb{R}$. Then,

$$E[aX + bY + c] = \underline{\hspace{4cm}}$$

<u>Exercises</u>

1. Suppose we have N items in a bag, K of which are successes. Suppose we draw (<u>without</u> replacement) until we have k successes, $k \leq K \leq N$. Let X be the number of draws until the k^{th} success. What is Ω_X? What is $p_X(n) = P(X = n)$? (We say X is a "negative hypergeometric" random variable).

2. A frog starts on a 1-dimensional number line at 0. At each second, independently, the frog takes a unit step right with probability p_1, to the left with probability p_2, and doesn't move with probability p_3, where $p_1 + p_2 + p_3 = 1$. After 2 seconds, let X be the location of the frog. Find the probability mass function for X, $p_X(k)$. Find $E[X]$. Find the probability mass function for $Y = |X|$, $p_Y(k)$, and $E[Y]$.

3. Suppose we have r independent random variables $X_1, \dots, X_r$ that each represent the number of coins flipped up to and including the first head, where $P(head) = p$. Recall that each X_i has probability mass function,

$$p_{X_i}(k) = P(X_i = k) = (1-p)^{k-1}p$$

a) What do you think $E[X_i]$ should be (without calculations) if $p = \frac{1}{2}$? If $p = \frac{1}{3}$? In the general case? (Proof in lecture next time.)

b) Suppose we define $X = X_1 + \cdots + X_r$. What does X represent, in English words? (Hint: think of performing each "trial" one after the other.)

c) What is Ω_X? Find the probability mass function for X, $p_X(k)$.

d) Find $E[X]$ using linearity of expectation.