CSE 312: Foundations of Computing II
Quiz Section #3: Conditional Probability

Review/Mini-Lecture/Main Theorems and Concepts From Lecture

Conditional Probability: $P(A \mid B) =$ ____________________

Independence: Events E and F are independent iff

$P(E \cap F) =$ ___________________, or equivalently

$P(F) =$ _____________ or $P(E) =$ _______________

Bayes Theorem: $P(A \mid B) =$ _____________________________

Partition: Nonempty events $E_1, \ldots, E_n$ partition the sample space Ω iff

- $E_1, \ldots, E_n$ are exhaustive: _______________________________________ , and
- $E_1, \ldots, E_n$ are pairwise mutually exclusive: _______________________________________
  - Note that for any event A (with $A \neq \emptyset$ and $A \neq \Omega$): ____ and ____ partition Ω

Law of Total Probability (LTP): Suppose $A_1, \ldots, A_n$ partition Ω and let B be any event. Then,

$P(B) =$ _______________________________ = ______________________________________

Bayes Theorem with LTP: Suppose $A_1, \ldots, A_n$ partition Ω and let A and B be events. Then,

$P(A \mid B) =$ _______________________________ = ______________________________________

Chain Rule: Suppose $A_1, \ldots, A_n$ are events. Then

$P(A_1 \cap \ldots \cap A_n) =$ ____

Exercises

1. Suppose we randomly generate a number from the positive integers $\{1, 2, 3, \dots\}$, and let A_k be the event we generate the number k, and suppose $P(A_k) = \left(\frac{1}{2}\right)^k$. Once we generate a number, suppose the probability that we win $\$j$ for $j = 1, \dots, k$ is uniform: $P(\text{win } \$j) = \frac{1}{k}$. Let B be the event we win exactly $\$1$. What is $P(A_1|B)$? (You may use the fact that $\sum_{j=1}^{\infty} \frac{1}{j \cdot a^j} = \ln\left(\frac{a}{a-1}\right)$ for $a > 1$).

2. Suppose there are three possible teachers to take CSE 312 from: Martin Tompa, Anna Karlin, and Larry Ruzzo. Suppose the ratio of grades $A: B: C: D: F$ for Martin's class is $1: 2: 3: 4: 5$, for Anna's class is $3: 4: 5: 1: 2$, and for Larry's class is $5: 4: 3: 2: 1$. Suppose you are assigned a grade randomly according to the given ratios when you take a class from one of these professors, irrespective of your performance. Furthermore, suppose Martin teaches your class with probability $\frac{1}{2}$ and Anna and Larry have an equal chance of teaching if Martin isn't. What is the probability you had Martin, given that you received an A? Compare this to the unconditional probability that you had Martin.

3. Suppose we have a coin with probability p of heads. Suppose we flip this coin n times independently. Let X be the number of heads that we observe. What is $P(X = k)$, for $k = 0, \dots n$? Verify that $\sum_{k=0}^{n} P(X = k) = 1$, as it should.

4. Suppose we have a coin with probability p of heads. Suppose we flip this coin until we flip a head for the first time. Let X be the number of times we flip the coin <u>up to and including</u> the first head. What is $P(X = k)$, for $k = 1,2, \dots$? Verify that $\sum_{k=1}^{\infty} P(X = k) = 1$, as it should. (You may use the fact that $\sum_{j=0}^{\infty} a^j = \frac{1}{1-a}$ for $|a| < 1$).