\[
\prod_{i=1}^{n-2} \left(1 - \frac{2}{n-i+1}\right) = \prod_{i=1}^{n-2} \frac{n-i-1}{n-i+1} \\
= \frac{2}{n} \cdot \frac{2}{n-1} \cdot \frac{3}{n+1} \cdot \frac{3}{n+2} \cdots \frac{3}{n-2} \cdot \frac{2}{n-3} \cdot \frac{1}{n-4}
\]

\[
= \frac{2}{n(n-1)} \leq \frac{2}{n^2}
\]

so the probability of error is \( < 1 - \frac{2}{n^2} \)

Run Karger's algorithm \( \frac{t n^2}{2} \) times, where

\( t \) is a parameter to be decided, each time

using independent choices of the contracted edges. At the end, output the smallest

of these \( t n^2/2 \) cuts.

The probability that \( C \) is not output in

any of these \( t n^2/2 \) iterations is

\[
\left(1 - \frac{2}{n^2}\right)^{t n^2/2} \leq \left(\frac{1}{e}\right)^t
\]

When \( t = 14 \), this is \( < 10^{-6} \)

This is simpler than the most efficient
deterministic algorithm based on network
flow.

\[
y = e^{-x} \\
1 - x \leq e^{-x} \quad \text{for all } x
\]

(\( (1-x)^{1/2} x \leq e^{-1} \))

Let \( x = \frac{2}{n^2} \)

\[
y = 1 - x
\]
8(a) \( X_1, X_2, \ldots, X_n \in \text{Unif}(0, \theta) \), \( \theta \) unknown

\[
L(X_1, X_2, \ldots, X_n | \theta) = \prod_{i=1}^{n} \frac{1}{\theta} = \frac{1}{\theta^n} = \theta^{-n}
\]

\[
\frac{d}{d \theta} L(X_1, \ldots, X_n | \theta) = -\frac{n}{\theta^{n+1}} = 0 \Rightarrow \hat{\theta} = +\infty
\]

\[ L = \frac{1}{\theta^n} \]

\[ \hat{\theta} = \max(X_1, \ldots, X_n) \]

(c) \( F(x) = P(\theta \leq x) = P(\max(X_1, \ldots, X_n) < x) \)

\[
= \left( \frac{x}{\theta} \right)^n
\]

\[
\frac{x}{\theta}
\]

\[ E[\hat{\theta}] = \frac{n}{n+1} \theta \]
#5 (b) \( \lambda_0 = 0.125, \ \lambda_1 = 0.5 \)

Let \( T \) be the time between the two ticket requests. \( \lambda = 1 \).

\[
P(A = 2, \mid T = 1) = \frac{P(T = 1, \lambda = 2) \cdot P(A = 2 \mid \lambda_0)}{P(T = 1, \lambda = 2) \cdot P(A = 2 \mid \lambda_0) + P(T = 1, \lambda = 2) \cdot P(A = 2 \mid \lambda_1)}
\]

\[
= \frac{P(0 < T < 1 \mid \lambda = 2)}{P(0 < T < 1 \mid \lambda_0) \cdot P(A = 2, \lambda_0) + P(0 < T < 1 \mid \lambda_1) \cdot P(A = 2, \lambda_1)}
\]

Using \( \delta = 0.5 \) gives an approximate, but not the exact answer.

#6 \( x_1, x_2, \ldots, x_{8761} \sim \text{Ber}(p) \), \( p \) unknown

\[
\hat{p} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad n = 8761
\]

\[
\text{Var}(x_i) = 0.25 = \sigma^2
\]

\( \sigma^2 \) is not \( \text{Var}(\hat{p}) \)

\[
\text{Var}(\hat{p}) = \text{Var} \left( \frac{1}{n} \sum_{i=1}^{n} x_i \right) = \frac{1}{n^2} \text{Var} \left( \sum_{i=1}^{n} x_i \right)
\]

\[
= \frac{1}{n^2} \sum_{i=1}^{n} \text{Var}(x_i) = \frac{1}{n^2} \cdot 8761 \cdot 0.25 = \frac{1}{n^2} \cdot n \cdot \sigma^2
\]

\[
\sigma^2 = \frac{\sigma^2}{n}
\]

If you use \( \text{Var}(\hat{p}) = \sigma^2 \), your \( \Delta = 1 \)