Supp worksheet, #6

P(crash in any day) = 0.1, indep. for each day.
P(≥ 87 crash-free days in next 100)
Let X = # of crash-free days in next 100
$X \sim Bin(100, 0.9)$, $E[X] = 100 \times 0.9 = 90$, $Var(X) = 90 \times .1 = 9$.

$$P(X \geq 87) = P(86.5 \leq X \leq 100.5)$$

$$= P\left(\frac{86.5-90}{3} \leq \frac{X-90}{3} \leq \frac{100.5-90}{3}\right)$$

$$\approx P\left(-1.17 \leq \frac{X-90}{3} \leq 3.5\right)$$

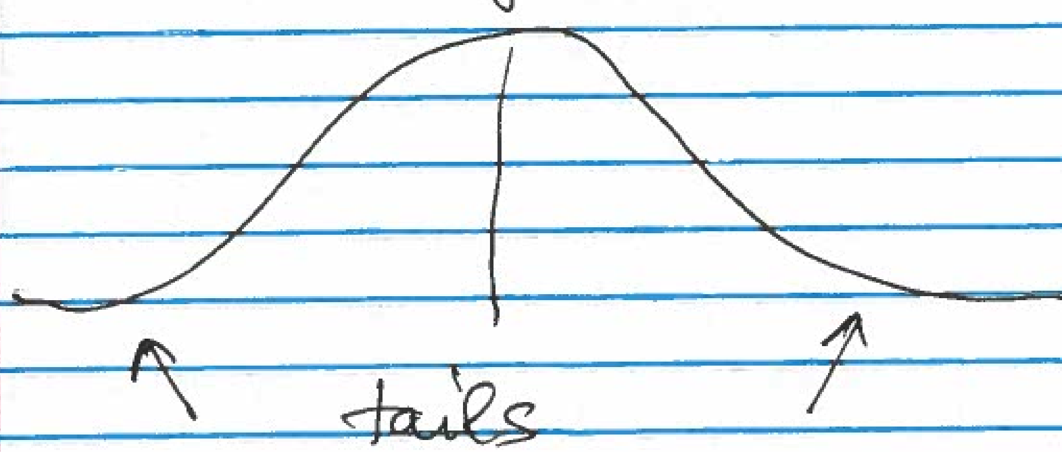
CLT $\rightarrow \approx \Phi(3.5) - \Phi(-1.17) = \Phi(3.5) + \Phi(1.17) - 1$

$$\approx \boxed{0.9998} + 0.8790 - \boxed{1} = 0.8788$$

Could we have simply used $P(X \geq 86.5)$

Tail Bounds: What is the probability of being far from the mean, particularly when you know very little about the distribution.

Ex: Your business's average daily expense is \$1500.
P(tomorrow's expense > \$6000).



Markov's Inequality

Theorem: If X is a nonnegative r.v., then for any $\alpha > 0$, $P(X \geq \alpha) \leq \frac{E[X]}{\alpha}$.

Equivalently, $P(X \geq kE[X]) \leq 1/k$, for $k > 0$

Ex: $P(\text{expense} \geq 6000) \leq \frac{1500}{6000} = 1/4$

Proof: $E[X] = \sum_x x P(X=x)$

$$= \sum_{x<\alpha} x P(X=x) + \sum_{x \geq \alpha} x P(X=x)$$

$$\geq 0 + \sum_{x \geq \alpha} \alpha P(X=x)$$

$$= \alpha P(X \geq \alpha)$$

So $\frac{E[X]}{\alpha} \geq P(X \geq \alpha)$

Chebyshev's inequality

Theorem: If Y is a r.v. with $E[Y] = \mu$, then for any $\alpha > 0$, $P(|Y - \mu| \geq \alpha) \leq \frac{Var(Y)}{\alpha^2}$.