\[ E[aX+b] = aE[X] + b \]
\[ E[X+Y] = E[X] + E[Y] \]
\[ E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\,dx \]
\[ Var(X) = E[(X-\mu)^2] = E[X^2] - (E[X])^2 \]
where \( \mu = E[X] \)

\( X \) and \( Y \) are independent iff
\[ \forall A \subset \mathbb{B} \quad P(X \in A \cap Y \in B) = P(X \in A)P(Y \in B) \]
(Discrete: \( X \) and \( Y \) ind iff \( \forall x \forall y \quad P(X=x \cap Y=y) = P(X=x)P(Y=y) \)

If \( X \) and \( Y \) are ind, \( Var(X+Y) = Var(X) + Var(Y) \)

**Special continuous random variables**

1. **Uniform:**
\( \begin{align*}
\& \quad X \sim \text{Uni}(\alpha, \beta) \quad f(x) = \begin{cases} 
\frac{1}{\beta-\alpha}, & \text{if } x \in [\alpha, \beta] \\
0, & \text{otherwise}
\end{cases} \\
\& \quad \mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{\alpha}^{\beta} \frac{x}{\beta-\alpha} \,dx = \frac{1}{\beta-\alpha} \int_{\alpha}^{\beta} x \,dx = \frac{1}{2} \left( \frac{\beta^2 - \alpha^2}{\beta-\alpha} \right) = \frac{\beta - \alpha}{2} \end{align*} \)
\[ E[X^2] = \int_{\alpha}^{\beta} x^2 \cdot \frac{1}{\beta - \alpha} \, dx = \frac{1}{\beta - \alpha} \cdot \left( \frac{2}{3} \beta^3 - \frac{2}{3} \alpha^3 \right) \]

\[ \frac{2}{3} \beta^3 - \frac{2}{3} \alpha^3 = \frac{\beta^3 - \alpha^3}{3 (\beta - \alpha)} = \frac{1}{3} (\alpha^2 + \alpha \beta + \beta^2) \]

\[ \text{Var}(X) = E[X^2] - (E[X])^2 = \frac{1}{3} (\alpha^2 + \alpha \beta + \beta^2) - \frac{1}{4} (\alpha \beta^2 + 2 \alpha^2 \beta + \beta^2) \]

\[ = \frac{1}{12} \beta^2 - \frac{1}{6} \alpha \beta + \frac{1}{12} eta^2 = \frac{1}{12} (\beta - \alpha)^2 = \frac{1}{12} (\beta - \alpha)^2 \]

2. Exponential distribution: random independent events happen at an average rate of \( \lambda \) per time unit. An exponential random variable is the time until the next event.

Ex: time until next particle emitted, time until next packet arrives at a server.

\[ X \sim \text{Exp}(\lambda) \]

\[ P(X > t) = e^{-\lambda t} \]

\[ F(t) = P(X < t) = 1 - e^{-\lambda t} \quad \text{for } t \geq 0, \text{ else } 0 \]

\[ f(x) = \frac{d}{dx} F(x) = \frac{d}{dx} (1 - e^{-\lambda x}) = (-e^{-\lambda x})(\lambda - 0) \]

\[ f(x) = \lambda e^{-\lambda x} \quad \text{for } x \geq 0, \text{ otherwise } f(x) = 0 \]

\[ E[X] = \frac{1}{\lambda} \]

\[ \text{Var}(X) = \frac{1}{\lambda^2} \]
Exponential is the continuous analog of the geometric distribution: both measure "time" until next event.

Poisson is number of events in a particular unit of time.
Exponential is time to the next event.

Memorylessness property of $X \sim \text{Exp}(\lambda)$:
$$P(X>s+t | X>s) = P(X>t), \text{ for any } s, t > 0.$$