Slide pack 6, slide 13:

Flip a coin with prob. p of heads n times independent

Let X be the number of heads.

$$p(i) = P(X=i) = P(i \text{ heads}) = \binom{n}{i} p^i (1-p)^{n-i}$$

$$E[X] = \sum_{i=0}^{n} i\,p(i) = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i}$$

This can be simplified, but how? Tune in Monday

Flip a coin with prob p of heads. Let X be the number of flips up to and including the first head, each flip independent of the others.

$$p(i) = P(X=i) = (1-p)^{i-1} p, \text{ for } i \in \{1, 2, \dots\}.$$

$$E[X] = \sum_{i=1}^{\infty} i\,p(i) = \sum_{i=1}^{\infty} i(1-p)^{i-1} p = p \sum_{i=1}^{\infty} i (1-p)^{i-1}.$$

Use calculus:

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x} = (1-x)^{-1} \quad \text{, where } |x| < 1$$

Take $\frac{d}{dx}$ of both sides:

$$\sum_{i=0}^{\infty} i x^{i-1} = +(1-x)^{-2} = \frac{1}{(1-x)^2}$$

$$\sum_{i=1}^{\infty} i x^{i-1} = \frac{1}{(1-x)^2}$$

Let $x = 1-p$:

$$E[X] = p \sum_{i=1}^{\infty} i (1-p)^{i-1} = \frac{p}{(1-(1-p))^2} = \frac{1}{p}$$

$p = \frac{1}{2} \Rightarrow E[X] = 2$

$p = \frac{1}{10} \Rightarrow E[X] = 10$