Schnapsen Club : CSE 503
3:30 – 5:30

Prior probability : $P(J) = 0.6$
Posterior probability: $P(J \mid F) \approx 0.871$

Ex: Paternity testing
Child has (A, a) genepair (event B_{Aa}). — genotype
Mother has (A, A)
Two possible fathers : $F_1 = (a,a)$, $F_2 = (A,a)$.
$P(F_1) = p$, $P(F_2) = 1-p$.

$$P(F_1 \mid B_{Aa}) = \frac{P(B_{Aa} \mid F_1)\, P(F_1)}{P(B_{Aa})}$$

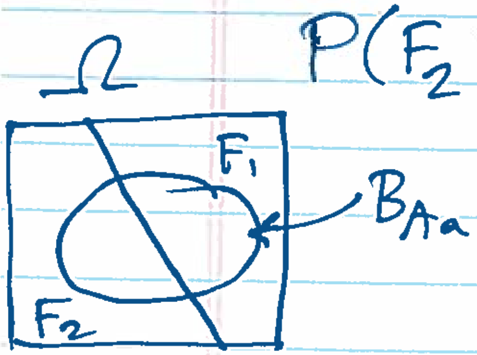
$$= \frac{P(B_{Aa} \mid F_1)\, P(F_1)}{P(B_{Aa} \mid F_1)\, P(F_1) + P(B_{Aa} \mid F_2)\, P(F_2)}$$

$$= \frac{1 \cdot p}{1 \cdot p + \frac{1}{2}(1-p)} = \frac{2p}{2p + (1-p)} = \frac{2p}{1+p}$$

How does this compare to the prior prob. p?

$$\frac{2p}{1+p} \geq \frac{2p}{1+1} = p$$

For instance, $p = \frac{1}{2} \Rightarrow \frac{2p}{1+p} = \frac{1}{3/2} = \frac{2}{3}$



$$P(F_2 \mid B_{Aa}) = 1 - P(F_1 \mid B_{Aa})$$
$$= 1 - \frac{2p}{1+p}$$

Defn: Two events E and F are *independent* iff $P(E \cap F) = P(E)P(F)$. Otherwise they are *dependent*.

Ex: Roll 2 fair 6-sided dice, yielding values D_1 and D_2.

Let $E = "D_1 = 1"$,
$F = "D_1 + D_2 = 7"$,
$G = "D_1 + D_2 = 5"$.

$P(E) = \frac{1}{6}$, $P(F) = \frac{6}{36} = \frac{1}{6}$

$P(E \cap F) = P(D_1 = 1 \cap D_2 = 6) = \frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6}$
$= P(E)P(F)$

so E and F are independent.

$P(G) = \frac{4}{36} = \frac{1}{9}$

$P(E \cap G) = P(D_1 = 1 \cap D_2 = 4) = \frac{1}{36} \neq \frac{1}{6} \cdot \frac{1}{9} = P(E)P(G)$

So E and G are dependent.

Defn: $E_1, E_2, \dots E_n$ are *independent* iff for every subset S of $\{1, 2, \dots, n\}$

$$P\left(\bigcap_{i \in S} E_i\right) = \prod_{i \in S} P(E_i).$$

Ex: Let X, Y each be ± 1 with equal probability and independently of each other.

$E = "X = 1"$, $F = "Y = 1"$, $G = "XY = 1"$.

Pairwise independent, but

$P(E \cap F \cap G) = P(E \cap F) = \frac{1}{4} \neq \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = P(E)P(F)P(G)$

Theorem: If $P(F) > 0$, then

E and F are independent iff $P(E|F) = P(E)$