Equally Likely Outcomes

\[ P(E) = \sum_{a \in E} P(a) \quad \text{(axiom 2)} \]

\[ = \sum_{a \in E} \frac{1}{12} = \frac{1E}{12} \]

Ex: Assume 2T face-up. Assume any 5-card hand is equally likely.

\[ P(\text{no trump in initial hand}) = \frac{\binom{13}{5}}{\binom{52}{5}} \]

\[ = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15} = \frac{3,003}{11,238} \approx 0.268 \]

Ex: Assume your 5-card hand is dealt before the trump is turned up.

\[ P(\geq 1 \text{ marriage in your hand}) = \frac{\binom{4}{1} \binom{13}{1} - \binom{4}{2} \binom{12}{1}}{\binom{52}{5}} \]

\[ \approx 0.204 \]

Ex: Assume 365 birthdays are equally probable. What is the prob. that, of n people, none share the same birthday?

Let \( E \) = assignment of a birthday to each of n people.

\[ E = \{ \text{unique today} \ldots \} \]

\[ P_n = P(\text{no shared birthday among n people}) \]

\[ = \frac{1E}{12} = \frac{P(365,n)}{365^n} = \frac{365!}{(365-n)!} \]

\[ = \frac{365 \cdot 364 \cdot 363 \ldots (365-n+1)}{365^n} \]
Some values:

\[ N = 23 \Rightarrow p_{23} < 0.5 \]
\[ N = 77 \Rightarrow p_{77} < 1/5000 \]
\[ N = 100 \Rightarrow p_{100} < 1/3 \times 10^{-5} \]

Conditional Probability of \( E \) given \( F \), written \( P(E \mid F) \), where \( F \neq \emptyset \), is the probability that \( E \) occurs, given that \( F \) occurred. Sample space \( \Omega \), event \( E \supseteq F \).

With equally likely outcomes,

\[
P(E \mid F) = \frac{|E \cap F|}{|F|} = \frac{P(E \cap F)}{P(F)}
\]

This turns out to be the formula even if outcomes aren’t equally likely.

Ex: Roll a fair die. What is \( P(S | \text{odd}) \)?

\( E = \{2, 5, 6\}, F = \{1, 3, 5\} \)

1. From counting: \( P(E \mid F) = \frac{|E \cap F|}{|F|} = \frac{|E|}{|F|} = \frac{1}{3} \)

2. From probabilities: \( P(E \cap F) = \frac{P(E \mid F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{1/6}{1/2} = \frac{1}{3} \)