Probability: reasoning under uncertainty
CSE examples:
1. Performance: failures uncertain, arrival rates and completion times uncertain
2. Patterns in data: Data mining, knowledge discovery.
   Netflix recommendations, Google page ordering, spam filter
3. Scientific data analysis: Uncertainty due to measurement errors.
4. Algorithm design: randomized algorithm

Motivation: If some probabilistic experiment has a finite set \( \Omega \) of equally likely outcomes (e.g., roll of a fair die), the probability of an event \( A \subseteq \Omega \) is
\[
P(A) = \frac{|A|}{|\Omega|} \quad \iff \text{ counting problems}
\]
Ex.: \( A = \{1, 3, 5\} \), \( \Omega = \{1, 2, 3, 4, 5, 6\} \).
\[
P(A) = \frac{3}{6} = \frac{1}{2}
\]
Ex.: Given a random 5-card hand, what is the probability it contains < 2 spades?
Product Rule:

If there are m choices for step 1 and for each choice, there are then n choices for step 2, there are mn choices in total.

Step 1
- m = 4

Step 2
- n = 3

Easily generalizes to 5 sequential steps.

Ex: How many n-bit strings are there? $2^n$

Ex: How many subsets does an n-element set have? $2^n$

For each element x of the set, there are 2 choices: either x is in the subset or it is not.