Review of Important Distributions

1. Discrete
2. Continuous

Discrete Random Variables

Discrete Uniform Distribution

Definition: A random variable that takes any integer value in an interval with equal likelihood

Example: Choose an integer uniformly between 0 and 10

Parameters: integers *a*, *b* (lower and upper bound of interval)

Notation: $X \sim \text{Unif}(a,b)$

Properties:

$$E[X] = \frac{a+b}{2}$$

$$\text{Var}(X) = \frac{(b-a)(b-a+2)}{12}$$

pmf: $P(X=k) = \frac{1}{b-a+1}$ for $k \in \{a, a+1, \ldots, b\}$

Bernoulli Distribution

Definition: value 1 with probability p, 0 with probability 1-p

Example: coin toss (p = ½ for fair coin)

Parameters: p

Notation: $X \sim \mathrm{Ber}(p)$

Properties:

$E[X] = p$

$\mathrm{Var}(X) = p(1-p)$

pmf: see Definition above

Binomial Distribution

Definition: sum of n independent Bernoulli trials, each with parameter p

Example: number of heads in 10 independent coin tosses

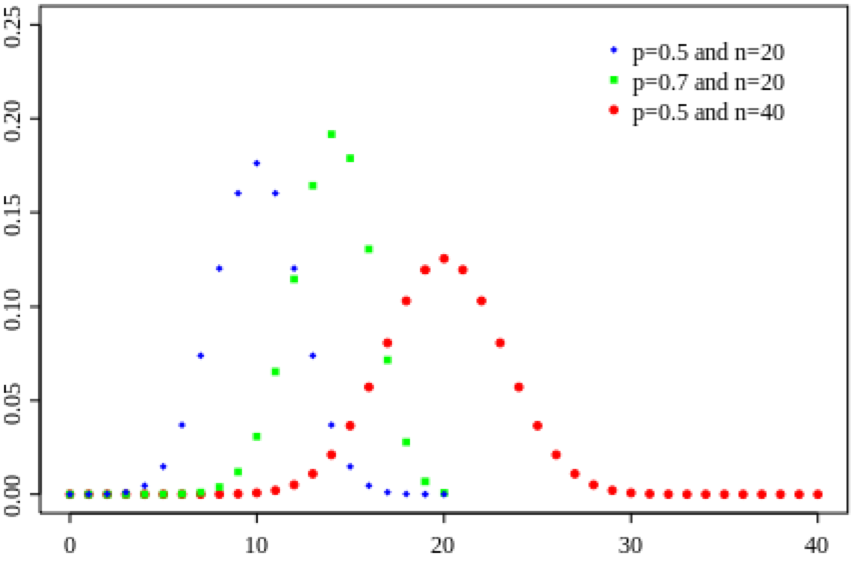
Parameters: n, p

Notation: $X \sim \text{Bin}(n, p)$

Properties:

$E[X] = np$

$\text{Var}(X) = np(1-p)$



pmf: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k \in \{0, 1, \ldots, n\}$

Geometric Distribution

Definition: number of independent Bernoulli trials with parameter *p* until and including first success (so *X* can take values 1, 2, 3, ...)

Example: # of coins flipped until first head

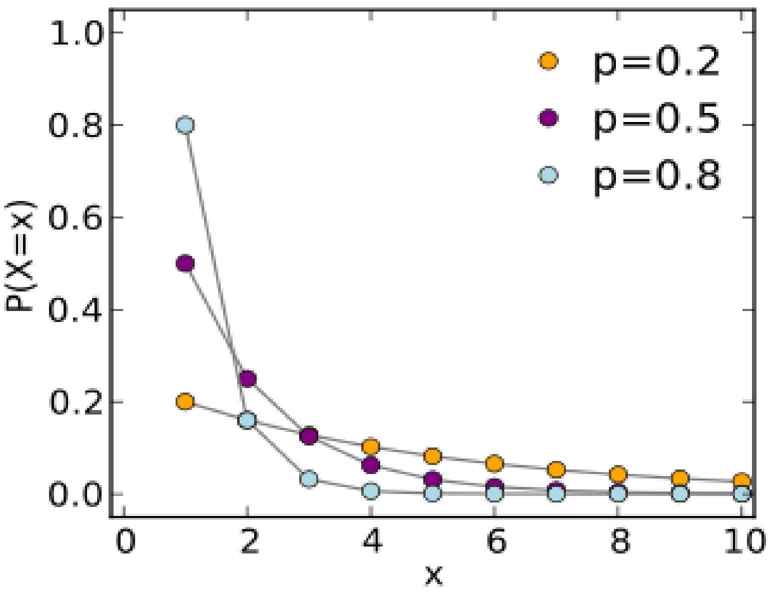
Parameters: *p*

Notation: $X \sim \text{geo}(p)$

Properties:

$E[X] = \frac{1}{p}$

$\text{Var}(X) = \frac{1-p}{p^2}$



pmf: $\mathrm{P}(X = k) = (1-p)^{k-1}p$ for $k \in \{1, 2, \ldots\}$

Hypergeometric Distribution

Definition: number of successes in *n* draws (without replacement) from *N* items that contain *K* successes in total

Example: An urn has 10 red balls and 10 blue balls. What is the probability of drawing 2 red balls in 4 draws?

Parameters: *n*, *N*, *K*

Properties:

E[X] = $n \cdot \frac{K}{N}$

Var(X) = $n \cdot \frac{K(N-K)(N-n)}{N^2(N-1)}$

pmf: $\mathrm{P}(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$

Think about the pmf; we've been doing it for weeks now: ways-to-choose-successes times ways-to-choose-failures divided by ways-to-choose-all.

Also, consider that the binomial dist. is the with-replacement analog of this.

Poisson Distribution

Definition: number of events that occur in a unit of time, if those events occur independently at an average rate λ per unit time

Example: # of cars at traffic light in 1 minute, # of deaths in 1 year by horse kick in Prussian cavalry

Parameters: λ

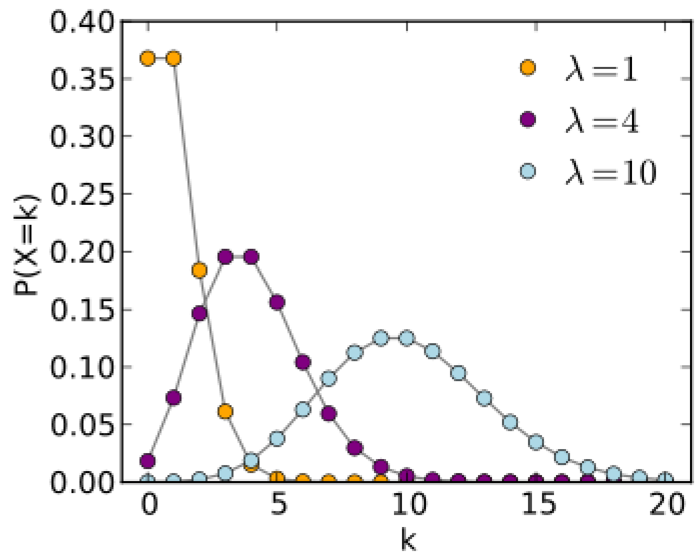
Notation: X ~ Poi(λ)

Properties:

E[X] = λ

Var(X) = λ

pmf: $\mathrm{P}(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$ for k ∈ {0, 1, …}



Continuous Random Variables

Continuous Uniform Distribution

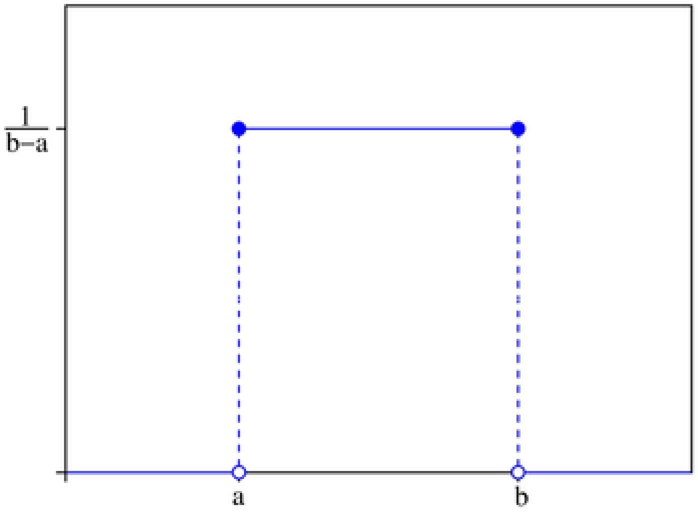
Definition: A random variable that takes any real value in an interval with equal likelihood

Example: Choose a real number (with infinite precision) between 0 and 10

Parameters: a, b (lower and upper bound of interval)

Notation: $X \sim \text{Uni}(a,b)$

Properties:



$$E[X] = \frac{a+b}{2}$$

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$

pdf: $f(x) = \frac{1}{b-a}$ if $x \in [a,b]$, 0 otherwise

Exponential Distribution

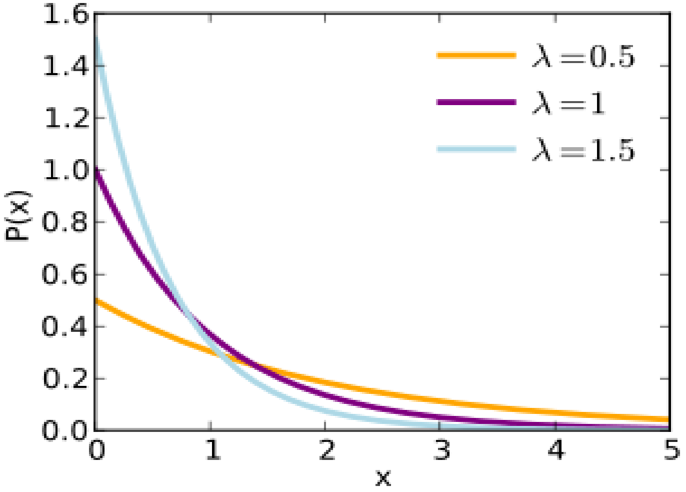
Definition: Time until next event in Poisson process

Example: How long until the next soldier is killed by horse kick?

Parameters: λ, the average number of events per unit time

Notation: $X \sim \text{Exp}(\lambda)$

Properties:



$\text{E}[X] = \frac{1}{\lambda}$

$\text{Var}(X) = \frac{1}{\lambda^2}$

pdf: $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$, $\quad$ 0 for $x < 0$

Normal Distribution

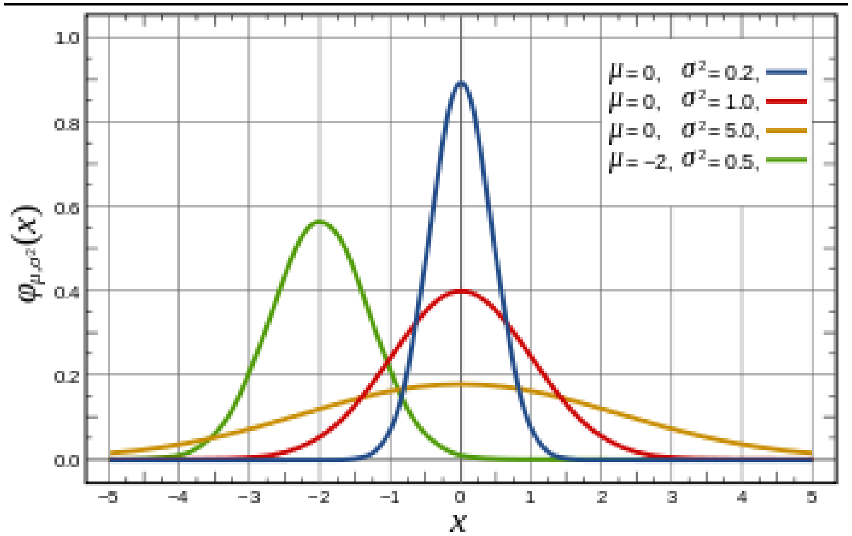
Description: Classic bell curve

Example: Quantum harmonic oscillator ground state (exact), Human heights, binomial random variables (approximate)

Parameters: μ, σ^2

Notation: $X \sim N(\mu, \sigma^2)$

Properties:



$E[X] = \mu$

$\mathrm{Var}(X) = \sigma^2$

pdf: $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}}\; e^{\frac{-(x-\mu)^2}{2\sigma^2}}$