Problem

Toss a red die and a green die. What is the probability that the sum mod 6 is 4 given that the green die shows a 5?

$$Pr((R+G) \bmod 6 = 4)|G=5) =$$

$$\frac{Pr(G=5 \text{ and } (R+G) \bmod 6 = 4)}{Pr(G=5)} = \frac{1}{6}$$

$$Pr(G=5 \text{ and } R=-1 \bmod 6) = Pr(G=5 \text{ and } R=5) = \frac{1}{36}$$

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Problem

In a group of N people 15% are left-handed.

Suppose that 100 times you pick a random person (each person is picked each time with probability 1/N) and ask that person if they are left-handed or not.

What is the probability that among the 100 queries, 55 people are left-handed?

$$\binom{100}{55}(0.15)^{55}(0.85)^{45}$$

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A lie detector is known to be 80% reliable when the person is guilty and 95% reliable when the person is innocent.

If a suspect is chosen from a group of suspects of which only 2% have ever committed a crime, and the test indicates that the person is guilty, what is the probability that he is innocent?

$I:$ event that he is innocent

$G:$ event that the test indicates guilty

$$Pr(I|G) = \frac{Pr(G|I)Pr(I)}{Pr(G)} = \frac{Pr(G|I)Pr(I)}{Pr(G|I)Pr(I) + Pr(G|I^c)Pr(I^c)}$$

$$= \frac{0.05 \cdot 0.98}{0.05 \cdot 0.98 + 0.8 \cdot 0.02}$$

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Problem

There is a population of N people. The number of good guys among these people is i with probability p_i

Take a sample of n people from the population. Each subset of n is equally likely. What is the probability that there are j good guys in the population conditioned on the fact that there are k good guys in the sample.

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Problem

There is a population of N people. The number of good guys among these people is i with probability p_i

Take a sample of n people from the population. What is the probability that there are j good guys in the population conditioned on the fact that there are k good guys in the sample.

$E_i =$ event that there are i good guys among N

$S_i =$ event that there are i good guys in sample

$$Pr(E_j|S_k) = \frac{Pr(S_k|E_j)Pr(E_j)}{Pr(S_k)} \qquad Pr(S_k|E_j) = \frac{\binom{j}{k}\binom{N-j}{n-k}}{\binom{N}{k}}$$

$$Pr(S_k) = \sum_j Pr(S_k|E_j)Pr(E_j) = \sum_j \frac{\binom{j}{k}\binom{N-j}{n-k}}{\binom{N}{k}} p_j$$

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