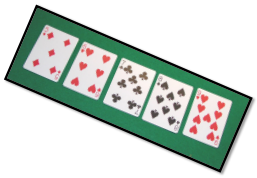
quick review of cards



- 52 total cards
- 13 different **ranks**: 2,3,4,5,6,7,8,9,10,J,Q,K,A
- 4 different **suits**: Hearts, Clubs, Diamonds, Spades

counting cards

- How many possible 5 card hands? $\binom{52}{5}$
- A "straight" is five consecutive rank cards of any suit. How many possible straights?

$10 \cdot 4^5 = 10,240$

- How many flushes are there?

$4 \cdot \binom{13}{5} = 5,148$

the sleuth's criterion (Rudich)

For each object constructed it should be possible to reconstruct the unique sequence of choices that led to it!

Example: How many ways are there to choose a 5 card hand that contains at least 3 aces?

$\binom{4}{3} \cdot \binom{49}{2}$ Choose 3 aces, then choose 2 cards from remaining 49.

the sleuth's criterion (Rudich)

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Example: How many ways are there to choose a 5 card hand that contains at least 3 aces?

~~$\binom{4}{3} \binom{49}{2}$~~ Choose 3 aces, then choose 2 cards from remaining 49.

$\binom{4}{3} \cdot \binom{48}{2} + \binom{4}{4} \cdot \binom{48}{1}$

When in doubt break set up into disjoint sets you **know** how to count!

- A, B, C, D, E, F, G (7 people) sitting in a row of 9 seats.
- How many ways to seat them?

9!/2!

- A, B, C, D, E, F, G sitting in a row of 9 seats.
- A, B, C, D must sit together in that order.
- How many ways to seat them?

6!/2!

Let H = ABCD
bijection between ordering of H, E, F, G on 6 seats and desired set

- A, B, C, D, E, F, G sitting in a row of 9 seats.
- A, B, C, D must sit together in that order.
- Also, E, F must sit together in that order.
- How many ways to seat them?

5!/2!

- A, B, C, D, E, F, G sitting in a row of 9 seats.
- A, B, C, D must sit together in that order.
- E and F must not sit together
- How many ways to seat these people?

6!/2! – 2(5!/2!)

Lessons

- Solve the same problem in different ways!
- If needed, break sets up into disjoint subsets that you know for sure how to count.
- Have in mind a sequence of choices that produces the objects you are trying to count. (Usually there are many possibilities.)
- Once you specify the sequence of choices you are making to construct the objects, make sure that given the result, you can tell exactly what choice was made at each step!