Recipe for finding pdf of g(X)

$Y = g(X)$

1) Find CDF

$$F_Y(y) = Pr(g(X) \leq y) = \int_{\{x|g(x)\leq y\}} f_X(x)dx$$

2) Differentiate to get pdf

$$f_Y(y) = \frac{d}{dy}F_Y(y)$$

Y = 2X.

Given the CDF and pdf of X, find the CDF and pdf of Y.

$X_1, X_2, X_3, \ldots X_n$ are i.i.d. U[0,1]

$X = \max(X_1, X_2, X_3, \ldots X_n)$. pdf of X?

Law of Total Probability

$$Pr(E) = \int_{-\infty}^{\infty} Pr(E|X = x) f_X(x)dx$$

Example: X, Y independent Uniform (0,1) r.v.s

$Pr(X < Y) =?$

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$Pr(X < Y) = ?$

$$Pr(X < Y) = \int_0^1 Pr(X < Y | X = x) f_X(x) dx$$
$$= \int_0^1 (1-x) f_X(x) dx \quad = \int_0^1 (1-x)dx$$
$$= \left(x - \frac{x^2}{2}\right)\Big|_0^1 = \frac{1}{2}$$

Joint distributions

Discrete	Continuous
$p_{X,Y}(x,y) = P(X=x, Y=y)$	$f_{X,Y}(x,y) \neq P(X=x, Y=y)$
$F_{X,Y}(x,y) = \sum_{t \le x} \sum_{s \le y} p_{X,Y}(t,s)$	$F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(t,s) ds dt$
$\sum_x \sum_y p_{X,Y}(x,y) = 1$	$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx dy = 1$
$p_X(x) = \sum_y p_{X,Y}(x,y)$	$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy$
$E[g(X,Y)] = \sum_x \sum_y g(x,y) p_{X,Y}(x,y)$	$E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y) dx dy$

Independence	$\forall x, y, p_{X,Y}(x,y) = p_X(x) p_Y(y)$	$\forall x, y, f_{X,Y}(x,y) = f_X(x) f_Y(y)$

Law of Total Expectation

$$E(Y) = \int_{-\infty}^{\infty} E(Y|X=x) f_X(x) dx$$

$$E(Y|X=x) = \int_{-\infty}^{\infty} y f_{Y|X}(y|X=x) dy$$

$$f_{Y|X}(y|X=x) = \frac{f_{XY}(x,y)}{f_X(x)} = f_{Y|X}(y|x)$$

Example

X is Uniform [0,10]

Y is Uniform on [0, X]

What is E(Y)? $\quad f_{XY}(x,y) = ? \quad f_Y(y) = ?$