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UW Schnapsen Club
Fridays 3:30–5:30
CSE 503

<u>Defn</u>: A <u>sample space</u> is a set Ω of possible "<u>outcomes</u>" of a probabistic experiment.

<u>Ex</u>: Flip of a coin: $\Omega = \{H, T\}$
2 flips of a coin: $\Omega = \{(H,H), (H,T), (T,H), (T,T)\}$
Roll a 6-sided die: $\Omega = \{1,2,3,4,5,6\}$
# of emails received in one day: $\Omega = \mathbb{N}$, the set of nonnegative integers
5-card hands dealt from a 52-card deck:
$|\Omega| = \binom{52}{5}$.

<u>Defn</u>: An <u>event</u> is any $E \subseteq \Omega$.

<u>Ex</u>: ≥ 1 head in 2 coin flips: $E = \{(H,H), (H,T), (T,H)\}$.
odd roll of die: $E = \{1,3,5\}$
5-card hand with no Spade: $|E| = \binom{39}{5}$.

<u>Defn</u>: E and F are <u>mutually exclusive</u> iff $E \cap F = \emptyset$.

<u>Defn</u>: There is a function P (called "probability") that assigns a real number P(E) to every event E satisfying the following 3 axioms:
(1) $P(E) \geq 0$
(2) $P(\Omega) = 1$
(3) If E and F are mutually exclusive, $P(E \cup F) = P(E) + P(F)$.

Implications of axioms:

(a) $P(\overline{E}) = P(\Omega - E) = 1 - P(E)$, because
$1 = P(\Omega) = P(E \cup \overline{E}) = P(E) + P(\overline{E})$

(b) If $E \subseteq F$, then $P(E) \le P(F)$, because
$P(F) = P(E \cup (F-E)) = P(E) + P(F-E)$
$\ge P(E) + 0 = P(E)$.

(c) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

(d) $P(E) \le 1$. (Follows from (b) and Axiom 2)

Start with the simple case of equally probable outcomes:
if $a \in \Omega$, $P(a) = 1/|\Omega|$.

Ex: Flip of a fair coin: $P(H) = P(T) = \frac{1}{2} = \frac{1}{|\Omega|}$.
2 flips of a fair coin: $P(HH) = \frac{1}{4}$
Roll of a fair die: $P(3) = 1/6$.

$$P(E) = \sum_{a \in E} P(a) = \sum_{a \in E} \frac{1}{|\Omega|} = \frac{|E|}{|\Omega|}.$$

Ex: $P(\text{5-card hand containing no Spade}) = \dfrac{\binom{39}{5}}{\binom{52}{5}}$
$$= \frac{39 \cdot 38 \cdot 37 \cdot 36 \cdot 35}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} \cdot \frac{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}$$
≈ 0.22

Ex: A "marriage" is the king and queen of the same suit.
$P(\text{5-card hand contains at least 1 marriage})$
$$= \frac{\binom{4}{1} \cdot \binom{50}{3} - \binom{4}{2}\binom{48}{1}}{\binom{52}{5}} \approx 2.5 \times 10^{-4}.$$

Ex: Assume 365 possible birthdays, and each is equally probable for a random person. What is the prob that, of n people, none share the same birthday?

$|\Omega| = 365^n$ ⟵ order matters

$|E| = \binom{365}{n}$? No.

$|E| = \frac{365!}{(365-n)!}$ ⟵ order matters

$P(\text{no shared birthday}) = \frac{365!}{(365-n)!\, 365^n}$

Some values:

$n = 23: P() < 0.5$

$n = 77: P() < 1/5000$

$n = 100: P() < 1/3\times 10^{15}$