Naïve Bayes Classifiers

Jonathan Lee and Varun Mahadevan

Programming Project: Spam Filter

Due: Thursday, November 10, 11:59pm

- Implement the Naive Bayes classifier for classifying emails as either spam or ham.
- You may use C, Java, Python, or R; ask if you have a different preference. We've provided starter code in Java and Python.
- Read Jonathan's notes, start early, and ask for help if you get stuck!

Spam vs. Ham

- (at least in the past) the bane of any email user's existence
- You know it when you see it!
- Easy for humans to identify, but not necessarily easy for computers
- Less of a problem for consumers now, because spam filters have gotten really good…

Dear Sir.

First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret. …

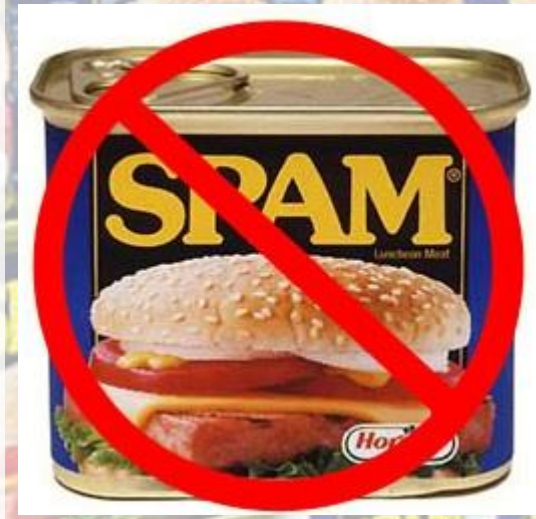
TO BE REMOVED FROM FUTURE MAILINGS, SIMPLY REPLY TO THIS MESSAGE AND PUT "REMOVE" IN THE SUBJECT.

99 MILLION EMAIL ADDRESSES FOR ONLY $99

Ok, Iknow this is blatantly OT but I'm beginning to go insane. Had an old Dell Dimension XPS sitting in the corner and decided to put it to use, I know it was working pre being stuck in the corner, but when I plugged it in, hit the power nothing happened.

The spam classification problem

- Input: collection of emails, already labelled *spam* or *ham*
  - Someone has to label these by hand!
  - Usually called the *training data*
- Use this data to train a model that "understands" what makes an email spam or ham
  - We're using a Naïve Bayes classifier, but there are other approaches
  - This is a *Machine Learning* problem (take 446 for more!)
- Test your model on emails whose label isn't known to the model, and see how well it does
  - Usually called the *test data*



Naïve Bayes in the real world

- One of the oldest, simplest models for *classification*
- Still, very powerful and used *all the time* in the real world/industry
  - Identifying credit card fraud
  - Identifying fake Amazon reviews
  - Identifying vandalism on Wikipedia
  - **Still** used (with modifications) by Gmail to prevent spam
  - Facial recognition
  - Categorizing Google News articles
  - Even used for medical diagnosis!

How do we represent an email?

- There's a lot of different things about emails that might give a computer a hint about whether or not it's spam
  - Possible *features*: words in body, subject line, sender, message header, time sent…
- For this assignment, we choose to represent an email just as the set of distinct words $(x_1, x_2, \ldots, x_n)$ in the subject and body

SUBJECT: Top Secret Business Venture

Dear Sir.

First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret…

(top, secret, business, venture, dear, sir, first, I, must, solicit, your, confidence, in, this, transaction, is, by, virture, of, its, nature, as, being, utterly, confidencial, and)

Notice that there are no duplicate words!

Naïve Bayes in theory

The concepts behind Naïve Bayes are nothing new to you -- we'll be using what we've learned in the past few weeks.

Specifically

- Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Law of Total Probability

$$P(A) = \sum_n P(A|B_n)P(B_n)$$

- Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Chain Rule

$$P(A_1, \ldots, A_n) = P(A_1)\, P(A_2|A_1) \ldots P(A_n|A_{n-1} \ldots A_1)$$

- Conditional Independence

$$P(A \cap B|C) = P(A|C)P(B|C)$$

- Take the set of distinct words $(x_1, x_2, \dots, x_n)$ to represent the text in an email.

- We are trying to compute

$$P(Spam|x_1, x_2, \dots, x_n) = ???$$

- By applying Bayes Theorem, we can reverse the conditioning. It's easier to find the probability of a word appearing in a spam email than the reverse.

$$P(Spam|x_1, x_2, \dots, x_n) = \frac{P(x_1, x_2, \dots x_n|Spam)P(Spam)}{P(x_1, x_2, \dots, x_n)}$$

$$= \frac{P(x_1, x_2, \dots x_n|Spam)P(Spam)}{P(x_1, x_2, \dots x_n|Spam)P(Spam) + P(x_1, x_2, \dots x_n|Ham)P(Ham)}$$

- Let's take a look at the numerator and apply the rule for Conditional Probability

$$P(x_1, x_2, \dots x_n|Spam)P(Spam) = P(x_1, x_2, \dots, x_n, Spam)$$

- And now let's use the Chain Rule to decompose this

$$P(x_1, x_2, \dots, x_n, Spam) = P(x_1|x_2, \dots, x_n, Spam)P(x_2, \dots, x_n, Spam)$$
$$= P(x_1|x_2, \dots, x_n, Spam)P(x_2|x_3, \dots, x_n, Spam)P(x_3, \dots, x_n, Spam)$$
$$= \cdots$$
$$= P(x_1|x_2, \dots, x_n, Spam) \dots P(x_{n-1}|x_n, Spam)P(x_n|Spam)P(Spam)$$

But this is still hard to compute.

- Let's simplify the problem with an assumption.
- We will assume that the words in the email are **conditionally independent of each other, given that we know whether or not the email is spam**.
- This is why we call this *Naïve* Bayes. **<u>This isn't true irl!</u>**
- So how does this help?

$$P(x_1, x_2, \dots, x_n, Spam)$$
$$= P(x_1|x_2, \dots, x_n, Spam) \dots P(x_{n-1}|x_n, Spam)P(x_n|Spam)P(Spam)$$
$$\approx P(x_1|Spam)P(x_2|Spam) \dots P(x_{n-1}|Spam)P(x_n|Spam)P(Spam)$$

$$P(x_1, x_2, \dots, x_n, Spam) \approx P(Spam) \prod_{i=1}^{n} P(x_i|Spam)$$

- So we know that

$$P(x_1, x_2, \ldots, x_n, Spam) \approx P(Spam) \prod_{i=1}^{n} P(x_i|Spam)$$

- Similarly

$$P(x_1, x_2, \ldots, x_n, Ham) \approx P(Ham) \prod_{i=1}^{n} P(x_i|Ham)$$

- Putting it all together

$$P(Spam|x_1, x_2, \ldots, x_n) \approx \frac{P(Spam) \prod_{i=1}^{n} P(x_i|Spam)}{P(Spam) \prod_{i=1}^{n} P(x_i|Spam) + P(Ham) \prod_{i=1}^{n} P(x_i|Ham)}$$

How spammy is a word?

- Have a nice formula for email spam probability, using conditional probabilities of words given ham/spam
- $P(Spam)$ and $P(Ham)$ are just the proportion of total emails that are spam and ham
- What is $P(viagra|Spam)$ asking?
- Would be easy to just count up how many spam emails have this word in them, so

$$P(w|Spam) = \frac{number\ of\ spam\ emails\ containing\ w}{total\ number\ of\ spam\ emails}$$ (maybe)

- This seems reasonable, but there's a problem…

- Suppose the word *Pokemon* only ever showed up in ham emails in the training data, never in spam

$$P(Pokemon|Spam) = 0$$

- Since the overall spam probability is the product of a bunch of individual probabilities, if any of those is 0, the whole thing is 0
- Any email with the word *Pokemon* would be assigned a spam probability of 0

SUBJECT: Get out of debt!

Cheap prescription pills! Earn fast cash using this one weird trick! Meet singles near you and get preapproved for a low interest credit card! Pokemon

definitely not spam, right?

- What can we do?

Laplace smoothing

- Crazy idea: what if we pretend we've seen every outcome once already?
- Pretend we've seen one more spam email *with* w, one more *without* w

$$P(w|Spam) = \frac{[number\ of\ spam\ emails\ containing\ w] + 1}{[total\ number\ of\ spam\ emails] + 2}$$

- Then, $P(Pokemon|Spam) > 0$
- No one word can "poison" the overall probability too much
- General technique to avoid assuming that unseen events will never happen