

CSE 312: Foundations of Computing II
Quiz Section #4: Expected value

1. Let the random variable X be the sum of two independent rolls of a fair die.
 - (a) What is the probability mass function of X ?
 - (b) From your answer to part (a), calculate $E[X]$.
2. Let the random variable X be the number of heads in n independent flips of a fair coin.
 - (a) What is the probability mass function of X ?
 - (b) From your answer to part (a), calculate $E[X]$.

Hint: prove and use the identity $i \binom{n}{i} = n \binom{n-1}{i-1}$.

3. This problem demonstrates that independence can be “broken” by conditioning. Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event “ $D_1 = 1$ ”, F be the event “ $D_2 = 6$ ”, and G be the event “ $D_1 + D_2 = 7$ ”. Even though E and F are independent, show that

$$P(E \cap F | G) \neq P(E | G) P(F | G).$$