Midterm Review

coverage

everything in text chapters 1-2, slides & homework pre-exam (except “continuous random variables,” started last week) is included, except as noted below.

mechanics

closed book; 1 page of notes (8.5 x 11, ≤ 2 sides, handwritten)

I’m more interested in setup and method than in numerical answers, so concentrate on giving a clear approach, perhaps including a terse English outline of your reasoning.

Corollary: calculators are probably irrelevant, but I will send email to the class list tomorrow with final word on whether they are (dis-)allowed counting principle (product rule)

permutations

combinations

indistinguishable objects

binomial coefficients

binomial theorem

partitions & multinomial coefficients

inclusion/exclusion

pigeon hole principle

sample spaces & events

axioms

complements, Venn diagrams, deMorgan, mutually exclusive events, etc.

equally likely outcomes

chapter 1: conditional probability and independence

conditional probability

chain rule, aka multiplication rule

total probability theorem

Bayes rule yes, learn the formula

odds (and prior/posterior odds form of Bayes rule)

independence

conditional independence

gambler's ruin

discrete random variables

probability mass function (pmf)

expectation of X

expectation of g(X) (i.e., a function of an r.v.)

linearity: expectation of X+Y and aX+b

variance

cumulative distribution function (cdf)

- cdf as sum of pmf from -∞

independence; joint and marginal distributions

important examples:

know pmf, mean, variance of these

- uniform, bernoulli, binomial, poisson, geometric

some important (discrete) distributions

Name	PMF	$E[k]$	$E[k^2]$	σ^2
Uniform(a,b)	$f(k) = \frac{1}{(b-a+1)}, k = a, a+1, \ldots, b$	$\frac{a+b}{2}$		$\frac{(b-a+1)^2-1}{12}$
Bernoulli(p)	$f(k) = \begin{cases} 1-p & \text{if } k=0 \\ p & \text{if } k=1 \end{cases}$	p	p	$p(1-p)$
Binomial(p,n)	$f(k) = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, 1, \ldots, n$	np		$np(1-p)$
Poisson(λ)	$f(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, \ldots$	λ	$\lambda(\lambda+1)$	λ
Geometric(p)	$f(k) = p(1-p)^{k-1}, k = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{2-p}{p^2}$	$\frac{1-p}{p^2}$
Hypergeometric(n,N,m)	$f(k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}, k = 0, 1, \ldots, N$	$\frac{nm}{N}$		$\frac{nm}{N}\left(\frac{(n-1)(m-1)}{N-1} + 1 - \frac{nm}{N}\right)$

See also the summary in B&T following pg 528

Calculus is a prereq, but I'd suggest the most important parts to brush up on are:

taylor's series for e^x

sum of geometric series: $\Sigma_{i\geq0}\ x^i = 1/(1-x)\ \ (0\leq x<1)$

Tip: multiply both sides by (1-x)

$\Sigma_{i\geq1}\ ix^{i-1} = 1/(1-x)^2$

Tip1: slide # ~13 in "random variables" lecture notes, or text

Tip2: if it were $\Sigma_{i\geq1}\ ix^{i+1}$, say, you could convert to the above form by dividing by x^2 etc.; 1st few terms may be exceptions

integrals & derivatives of polynomials, e^x;

chain rule for derivatives; integration by parts

Good Luck!