CSE 312: Foundations of Computing II
Quiz Section #4

1. Corrupted by their power, the judges running the popular game show America's Next Top Mathematician have been taking bribes from many of the contestants. During each of two episodes, a given contestant is either allowed to stay on the show or is kicked off. If the contestant has been bribing the judges, she will be allowed to stay with probability 1. If the contestant has not been bribing the judges, she will be allowed to stay with probability 1/3, independent of what happens in earlier episodes. Suppose that 1/4 of the contestants have been bribing the judges. The same contestants bribe the judges in both rounds.
   (a) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she was bribing the judges?
   (b) If you pick a random contestant, what is the probability that she is allowed to stay during both episodes?
   (c) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?
2. Let the random variable X be the sum of two independent rolls of a fair die.
   (a) What is the probability mass function of X?
   (b) From your answer to part (a), calculate $\mathrm{E}[X]$.
3. Let the random variable X be the number of heads in n independent flips of a fair coin.
   (a) What is the probability mass function of X?
   (b) From your answer to part (a), calculate $\mathrm{E}[X]$.
   Hint: prove and use the identity $i\binom{n}{i} = n\binom{n-1}{i-1}$.
4. This problem demonstrates that independence can be "broken" by conditioning. Let D_1 and D_2 be the outcomes of two independent rolls of a fair die. Let E be the event "$D_1 = 1$", F be the event "$D_2 = 6$", and G be the event "$D_1 + D_2 = 7$". Even though E and F are independent, show that
$$\mathrm{P}(E \cap F \mid G) \neq \mathrm{P}(E \mid G)\,\mathrm{P}(F \mid G).$$