CSE312 Quiz Section: January 10, 2013

1. How many ways are there to select 5 cards from a standard deck of 52 cards, where the 5 cards contain cards from at most two suits if:

   (a) order does not matter

   (b) order matters

2. Consider all natural numbers between 2 and 18 (both included).

   (a) How many of them are prime?

   (b) How many of them are prime or of the form $3k - 1, k \in N$?

   (c) How many are prime, even or of the form $2^l + 1$, where l is a natural number or zero?

   Hint: Inclusion-Exclusion

3. Consider a set of 25 people that form a social network. (The structure of the social network is determined by which pairs of people in the group are "friends".) How many possibilities are there for the structure of this social network?

4. At a dinner party, all of the n people present are to be seated at a circular table. Suppose there is a nametag at each place at the table and suppose that nobody sits down in their correct place. Show that it is possible to rotate the table so that at least two people are sitting in the correct place.

5. Show that:

   (a) $i^2 = 2\binom{i}{2} + \binom{i}{1}$

   (b) $i^3 = 6\binom{i}{3} + 6\binom{i}{2} + \binom{i}{1}$

6. Fix a non-negative integer r. Prove that $\sum_{i=0}^{n} \binom{i}{r} = \binom{n+1}{r+1}$.

7. Use (5) and (6) to show that:

$$\sum_{i=0}^{n} i^2 = 2\binom{n+1}{3} + \binom{n+1}{2}$$

   Expand to prove the familiar identity:

$$\sum_{i=0}^{n} i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$