

## Stuff you should know!

1. Multiplicative rule for counting. ( $m_1$  choices for the first item,  $m_2$  choices for the second...) Applications: number of words from alphabet of size  $k$ , and number of binary strings of length  $k$ . (explain!)

2. Additive rule for counting. (if Sets are disjoint, you can add them separately.) Prove that

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

3. Inclusion exclusion. For two sets, and also for  $n$  sets.

4. Pigeonhole principle. What if you have  $n$  pigeonholes and  $2n + 1$  pigeons? What if  $kn + 1$  for any  $k \geq 1$ ?

5. Binomial Theorem. Used to prove  $\sum_{j=0}^n (-1)^j \cdot \binom{n}{j} = 0$ . Can you prove this using an expectation, where you are picking a random binary string of length  $n$ , and win +1 dollar if it has an even number of ones, and lose -1 dollar if it has an odd number of ones?

6. Five-card poker hands are great practice! (i) one pair, (ii) two pair, (iii) three of a kind, (iv) straight, (v) flush, (vi) one card of each suit, (vii) four of a kind, (viii) full house. *Count 'em!*

7. Counting Two Ways.  $\binom{n+1}{k+1} = \sum_{i=0}^n \binom{i}{k}$ , and  $i^2 = \binom{i}{1} + 2\binom{i}{2}$ , and  $i^3 = 6\binom{i}{3} + 6\binom{i}{2} + \binom{i}{1}$

8. Probability identities. (\*\*) axioms, (i) inclusion-exclusion, (ii) 1-complement, (iii) independence, (iv) disjoint events, (v) sum to one, (vi) law of total probability, (vii) conditional expectation, (vi+vii) Bayes theorem.

9. Definition of a random variable, as a function. And also of  $g(X)$ . And interpretation of  $\{X = a\}$  as an event.

10. Joint random variables. Marginal probability. Conditional expectation.

11. Linearity of expectation. Used to prove  $\mathbf{E}$  of binomial. Other uses?

12. Two formulas for variance.  $\text{var}(X) = \mathbf{E}[(X - \mathbf{E}[X])^2] = \mathbf{E}[X^2] - \mathbf{E}[X]^2$ . Why is it always  $\geq 0$ ?

13. Roll two six sided and fair dice. Let  $X$  = sum of the dice. What is the pmf of  $X$ ? What is  $\mathbf{E}[X]$ ? What is  $\text{var}[X]$ ? Let  $Y = X \pmod{2}$ . What is the pmf of  $Y$ ? What is  $\mathbf{E}[X \mid Y = 0]$ ? What is  $\mathbf{E}[X \mid Y = 1]$ ?

14. **Bernoulli** Random Variable. Parameter  $p$ . Sample space. pmf. Expectation. Variance.
15. **Binomial** Random Variable. Parameters  $n, p$ . Sample space. pmf. Expectation. Variance (*use example 1 below*).
16. **Poisson** Random Variable. Parameter  $\lambda$ . Sample space. pmf. Expectation. Variance (tricky).
17. **Geometric** Random Variable. Parameter  $p$ . Sample space. pmf. Expectation. Variance (tricky).
18. **Discrete Uniform** Random Variable. Sample space. pmf/Expectation/Variance (**think about these**).