3: More counting + pigeonhole principle

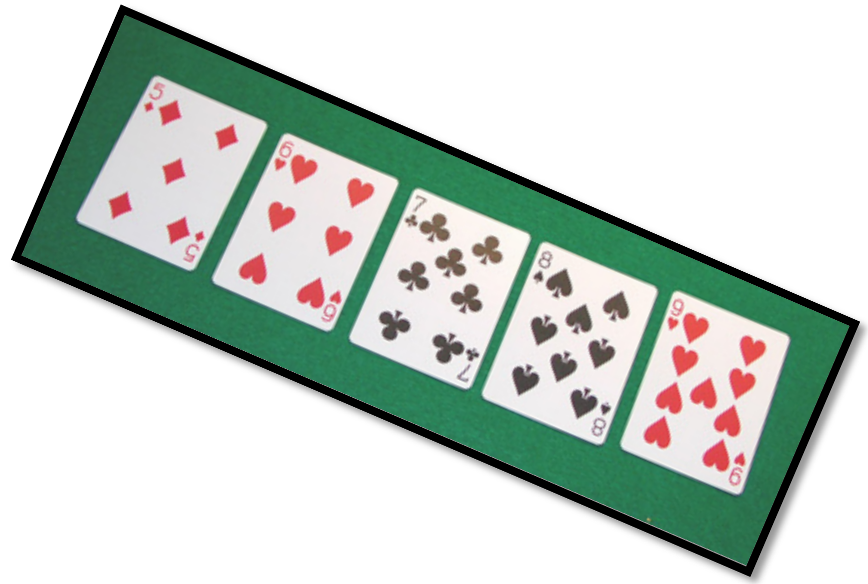
BT Section 1.6, Rosen, Section 7.5 Inclusion-exclusion

Permutations: Number of ways to order n distinct objects.

$$n! = n \cdot (n-1) \cdots 2 \cdot 1$$

Combinations: Number of ways to choose r things from n things

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$



- 52 total cards
- 13 different **ranks**:

2,3,4,5,6,7,8,9,10,J,Q,K,A

- 4 different **suits**: Hearts, Clubs, Diamonds,

Spades

For each object constructed it should be possible to reconstruct the unique sequence of choices that led to it!

Example: How many ways are there to choose a 5 card hand that contains at least 3 aces?

~~$\binom{4}{3} \cdot \binom{49}{2}$~~ ~~Choose 3 aces, then choose 2 cards from remaining 49.~~

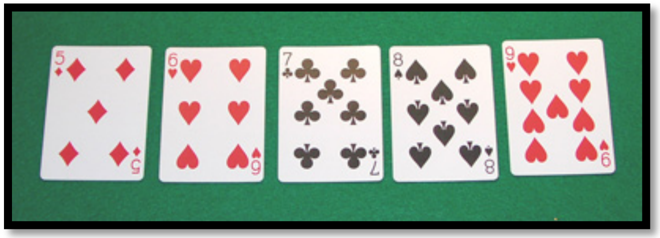
$$\binom{4}{3} \cdot \binom{48}{2} + \binom{4}{4} \cdot \binom{48}{1}$$

- How many possible 5 card hands?

$$\binom{52}{5}$$

- A "straight" is five consecutive rank cards of any suit. How many possible straights?

$$10 \cdot 4^5 = 10,240$$



- How many flushes are there?

$$4 \cdot \binom{13}{5} = 5,148$$

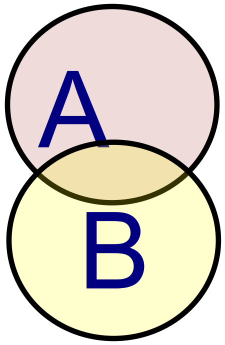
- How many straights that are not flushes?

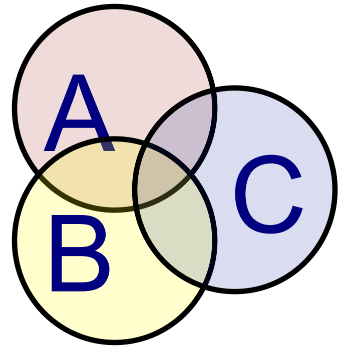
$$10 \cdot 4^5 - 10 \cdot 4 = 10,200$$

- How many flushes that are not straights?

$$4 \cdot \binom{13}{5} - 10 \cdot 4 = 5,108$$

inclusion/exclusion principle





$|A \cup B| =$
$|A|+|B|-|A \cap B|$

$|A \cup B \cup C| =$
$|A| + |B| + |C|$
$-|A \cap B|-|A \cap C|-|B \cap C|$
$+ |A \cap B \cap C|$

General: + singles - pairs + triples - quads + ...

- How many hands have at least three cards of one rank (**three of a kind**)?

$$13 \cdot \binom{4}{3} \cdot \binom{48}{2} + 13 \cdot 48 = 59,280$$

- How many hands are straights or flushes or three of a kind?

Inclusion/exclusion:

Flushes + Straights + 3OfKind
– (Flushes AND Straights) – (Flushes AND 3OfKind) – (Straights AND 3OFKind)
+ (Flushes AND Straights AND 3OfKind)

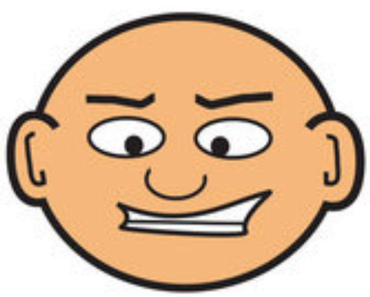
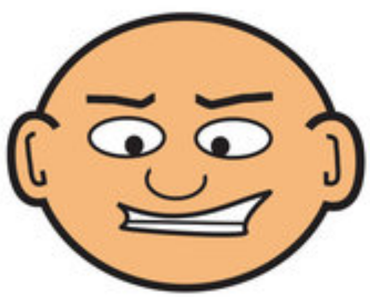
pigeonhole principle

If there are **n** pigeons in **k** holes and **n > k**, then **some hole contains more than one pigeon**.

More precisely, some hole contains at least $\lceil n/k \rceil$ pigeons.

There are two people in London who have the same number of hairs on their head.

- Typical head ~ 150,000 hairs
- Let's say max-harry-head ~ 1,000,000 hairs
- Since there are more than 1,000,000 people in London…

friending pigeons

There are many people in this room, some of whom are friends, some of whom are not…

Prove that some two people have the same number of friends.

Pigeonhole principle:

There are N people and only N-**1** different friend counts. (Cannot have a person with 0 AND a person with N-**1** friends!)