

Final Review

Coverage—comprehensive, slight emphasis post-midterm

pre-mid: B&T ch 1-2

post-mid: B&T ch 3,5,9, continuous, limits, hypothesis testing, mle, em.

everything in slides, hw, non-supl reading on “Schedule & Reading” web page

Mechanics

closed book, aside from one page of notes (8.5 x 11, both sides, handwritten)

I'm more interested in setup and method than in numerical answers, so concentrate on giving a clear approach, perhaps including a terse english outline of your reasoning.

Corollary: calculators are probably irrelevant, but bring one to the exam if you want, just in case.

Format—similar to midterm:

T/F, multiple choice, problem-solving, explain, ...

Story problems

see midterm review slides

chapter 3: continuous random variables

especially 3.1–3.3; light coverage: 3.4–3.6

probability density function (pdf)

cdf as integral of pdf from $-\infty$

expectation and variance

why does variance matter? a simple example: a random X arrives at a server, and chews up $f(X)$ seconds of CPU time. If $f(x)$ is a quadratic or cubic or exponential function, then randomly sampled X 's in the right tail of the distribution can greatly inflate average CPU demand even if rare, so variance (and, more generally, the shape of the distribution) matters a lot, even for a fixed mean. Recall, in general, $E[f(X)] \neq f(E[X])$.

important examples know pdf and/or cdf, mean, variance of these

uniform, normal (incl Φ , “standardization”), exponential

tail bounds

Markov

Chebyshev

Chernoff (lightly)

limit theorems

weak/strong laws of large numbers

central limit theorem

moment generating functions

lightly - see 2-3 slides in “limits” section; skim B&T 4.9 for more

likelihood, parameter estimation, MLE (b&t 9.1)

likelihood

“likelihood” of observed data given a model

usually just a product of probabilities (or densities: “ $\lim_{\delta \rightarrow 0} \dots$ ”), by independence assumption

a function of (unknown?) parameters of the model

parameter estimation

if you know/assume the *form* of the model (e.g. normal, poisson,...), can you estimate the *parameters* based on observed data

many ways

maximum likelihood estimators

one way to do it—choose values of the parameters that maximize likelihood of observed data

method (usually) – solve

“derivative (wrt parameter/s) of (log) likelihood = 0”

expectation maximization (EM)

EM

iterative algorithm trying to find MLE in situations that are analytically intractable

usual framework: there are 0/1 *hidden variables* (e.g., from which component was this datum sampled) & problem much easier if they were known

E-step: given rough parameter estimates, find expected values of hidden variables

M-step: given expected values of hidden variables, find (updated) parameter estimates to maximize likelihood

Algorithm: iterate above alternately until convergence

hypothesis testing (b&t 9.3)

I have data, and 2 hypotheses about the process generating it. Which hypothesis is (more likely to be) correct?

Again, a very rich literature on this. Here consider the case of 2 *simple* hypotheses, e.g. $p = 1/2$ vs $p = 2/3$

One of the many approaches: the “Likelihood Ratio Test”

calculate: $\frac{\text{likelihood of data under } \textit{alternate} \text{ hypothesis } H_1}{\text{likelihood of data under } \textit{null} \text{ hypothesis } H_0}$

ratio > 1 favors alternate, < 1 favors null, etc.

type 1, type 2 error, α , β , etc. Of special interest: α = “significance” - prob of falsely rejecting null when it’s true.

significance testing (b&t 9.4)

As above:

I have data, and 2 hypotheses about the process generating it. Which hypothesis is (more likely to be) correct?

But, consider *composite* hypotheses, e.g., $p = 1/2$ vs $p \neq 1/2$.

Can't do likelihood for composite, so no LRT

But can often still evaluate *significance*: devise a summary statistic whose distribution you can calculate *under the null*, so you can estimate probability seeing data that cause you to falsely rejecting the null when it's true. [Very often the stat follows normal- or t-distribution. Thank you, CLT.]

p-values: smallest α allowing rejection; probability of generating this data *assuming the null is true*, not the probability that the null is false. [Note that "Null=T/F" is usually not a probabilistic question, so "prob that null is F" is a nonsensical statement".]

Noise, uncertainty & variability are pervasive

Learning to model it, derive knowledge, and compute despite it are critical

E.g., knowing the mean is valuable, but two scenarios with the same mean and different variances can behave very differently in practice.

Stat 390/1 probability & statistics
CSE 427/8 computational biology
CSE 440/1 human/computer interaction
CSE 446 machine learning
CSE 472 computational linguistics
CSE 473 artificial intelligence

and others!

Thanks and Good Luck!



Hell's library

what to expect
on the final in
more detail

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