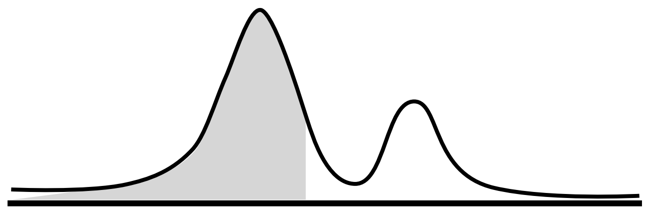
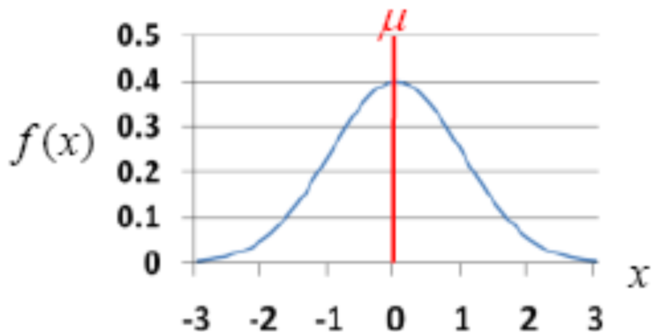
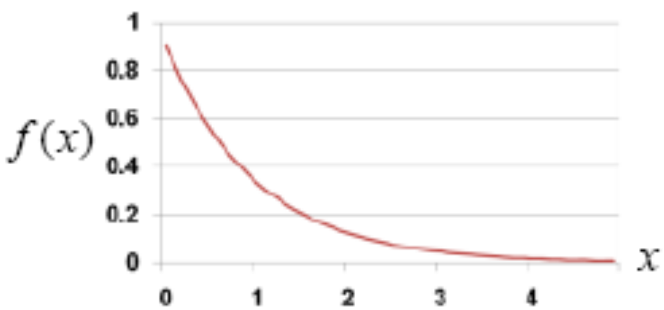
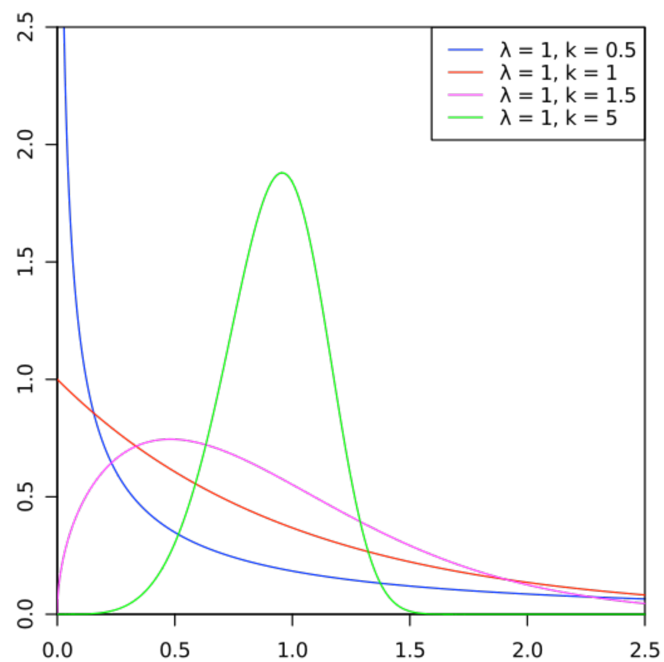




7. continuous random variables





Discrete random variable: takes values in a finite or countable set, e.g.

$X \in \{1,2, \ldots, 6\}$ with equal probability

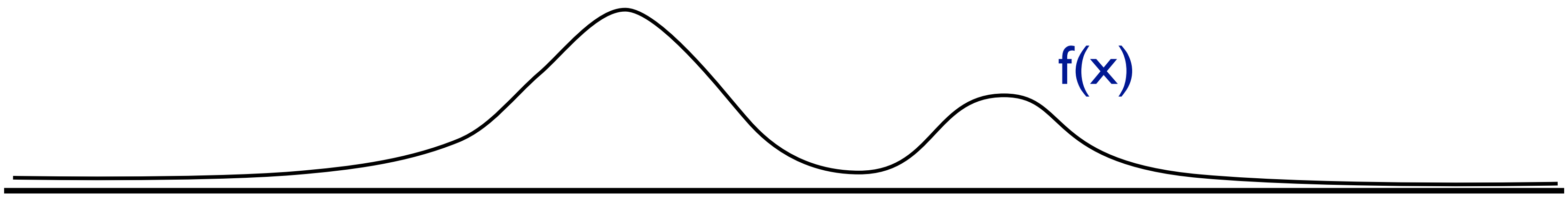
X is positive integer i with probability 2^{-i}

Continuous random variable: takes values in an uncountable set, e.g.

X is the weight of a random person (a real number)

X is a randomly selected point inside a unit square

X is the waiting time until the next packet arrives at the server f(x): R→R, the *probability density function* (or simply "density")



Require:

$f(x) \geq 0$, and

$\int_{-\infty}^{+\infty} f(x)\, dx = 1$

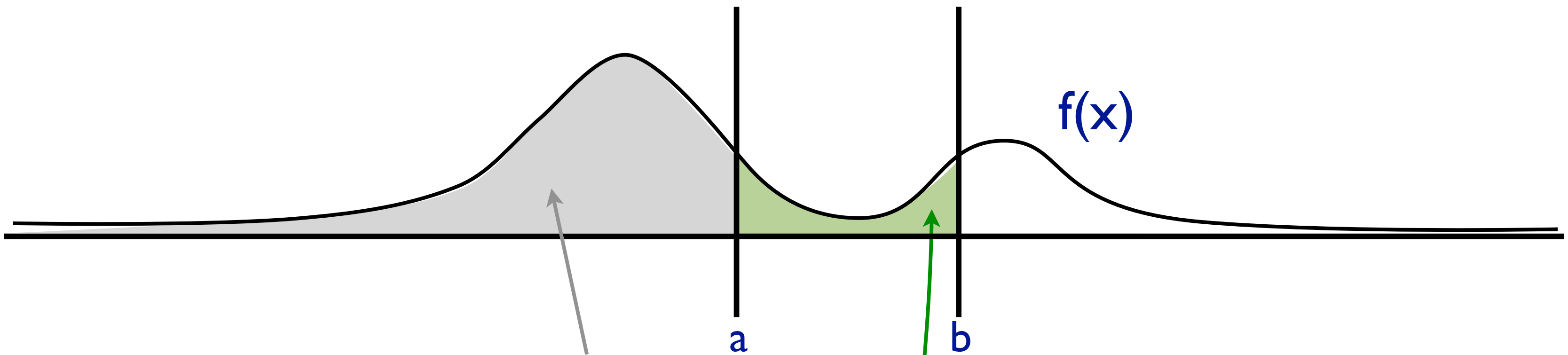
I.e., distribution is:

← nonnegative, and

← normalized,

just like discrete PMF

F(x): the *cumulative distribution function* (aka the "distribution")



$F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)\, dx$ (Area left of a)

$P(a < X \leq b) = F(b) - F(a)$ (Area between a and b)

A key relationship:

$f(x) = \frac{d}{dx} F(x)$, since $F(a) = \int_{-\infty}^{a} f(x)\, dx$, Densities are *not* probabilities; e.g. may be > 1

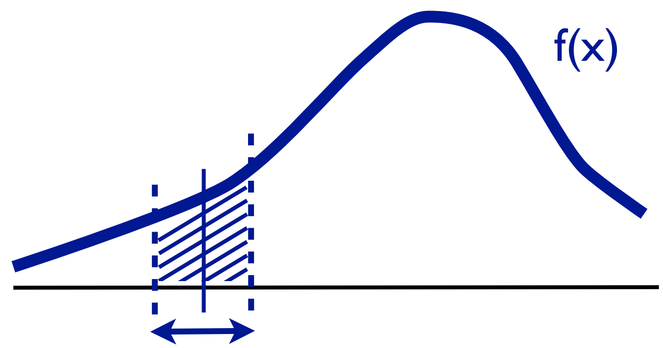
$P(X = a) = \lim_{\varepsilon \to 0} P(a-\varepsilon < X \le a) = F(a)-F(a) = 0$

I.e., the probability that a continuous random variable falls *at* a specified point is *zero*

$P(a - \varepsilon/2 < X \le a + \varepsilon/2) =$

$F(a + \varepsilon/2) - F(a - \varepsilon/2)$

$\approx \varepsilon \cdot f(a)$



I.e., The probability that it falls *near* that point is proportional to the density; in a large random sample, expect more samples where density is higher (hence the name "density").

Much of what we did with discrete r.v.s carries over almost unchanged, with Σ_x... replaced by $\int$... dx

E.g.

For discrete r.v. X, $E[X] = \Sigma_x \, xp(x)$

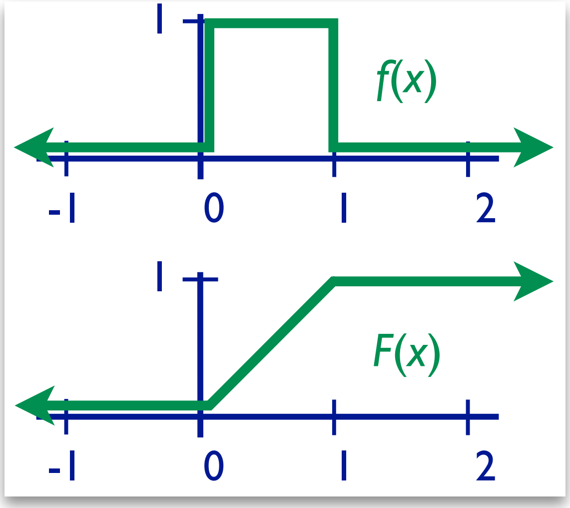
For continuous r.v. X, $E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

Why?

(a) We define it that way

(b) The probability that X falls "near" x, say within $x \pm dx/2$, is $\approx f(x)dx$, so the "average" X should be $\approx \Sigma\, xf(x)dx$ (summed over grid points spaced dx apart on the real line) and the limit of that as $dx \to 0$ is $\int xf(x)dx$

Let $f(x) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$



$$
\begin{aligned}
F(a) &= \int_{-\infty}^{a} f(x)dx \\
&= \begin{cases} 0 & \text{if } a \le 0 \\ a & \text{if } 0 < a \le 1 \text{ (since } a = \int_0^a 1dx) \\ 1 & \text{if } 1 < a \end{cases} \\
E[X] &= \int_{-\infty}^{\infty} x f(x)dx = \int_0^1 x\, dx = \left.\frac{x^2}{2}\right|_0^1 = \frac{1}{2} \\
E[X^2] &= \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^1 x^2\, dx = \left.\frac{x^3}{3}\right|_0^1 = \frac{1}{3}
\end{aligned}
$$

$$\text{Var}[X] = E[X^2] - (E[X])^2 = \tfrac{1}{3} - \tfrac{1}{4} = \tfrac{1}{12} \quad (\sigma \approx 0.29)$$

Linearity

$$E[aX+b] = aE[X]+b$$

$$E[X+Y] = E[X]+E[Y]$$

still true, just as for discrete

Functions of a random variable

$$E[g(X)] = \int g(x)f(x)dx$$

just as for discrete, but w/integral

Alternatively, let $Y = g(X)$, find the density of Y, say f_Y, (see B&T 4.1; somewhat like r.v. slides 33-35) and directly compute $E[Y] = \int y f_Y(y)dy$.

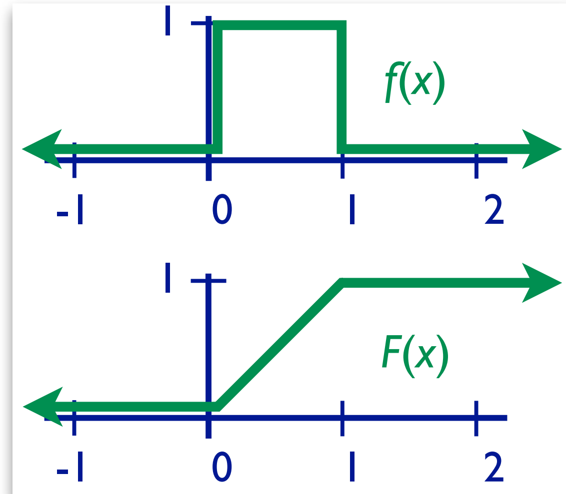
Definition is same as in the discrete case

$\mathrm{Var}[X] = E[(X-\mu)^2]$ where $\mu = E[X]$

Identity still holds:

$\mathrm{Var}[X] = E[X^2] - (E[X])^2$ proof "same"

Let $f(x) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$



$$
\begin{aligned}
F(a) &= \int_{-\infty}^{a} f(x)dx \\
&= \begin{cases} 0 & \text{if } a \le 0 \\ a & \text{if } 0 < a \le 1 \text{ (since } a = \int_0^a 1dx) \\ 1 & \text{if } 1 < a \end{cases}
\end{aligned}
$$

$$E[X] = \int_{-\infty}^{\infty} x f(x)dx = \int_0^1 x\, dx = \left.\frac{x^2}{2}\right|_0^1 = \frac{1}{2}$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^1 x^2\, dx = \left.\frac{x^3}{3}\right|_0^1 = \frac{1}{3}$$

$$\mathrm{Var}[X] = E[X^2] - (E[X])^2 = \tfrac{1}{3} - \tfrac{1}{4} = \tfrac{1}{12} \quad (\sigma \approx 0.29)$$

Continuous random variable X has density f(x), and

$$\Pr(a \le X \le b) = \int_a^b f(x)\,dx$$

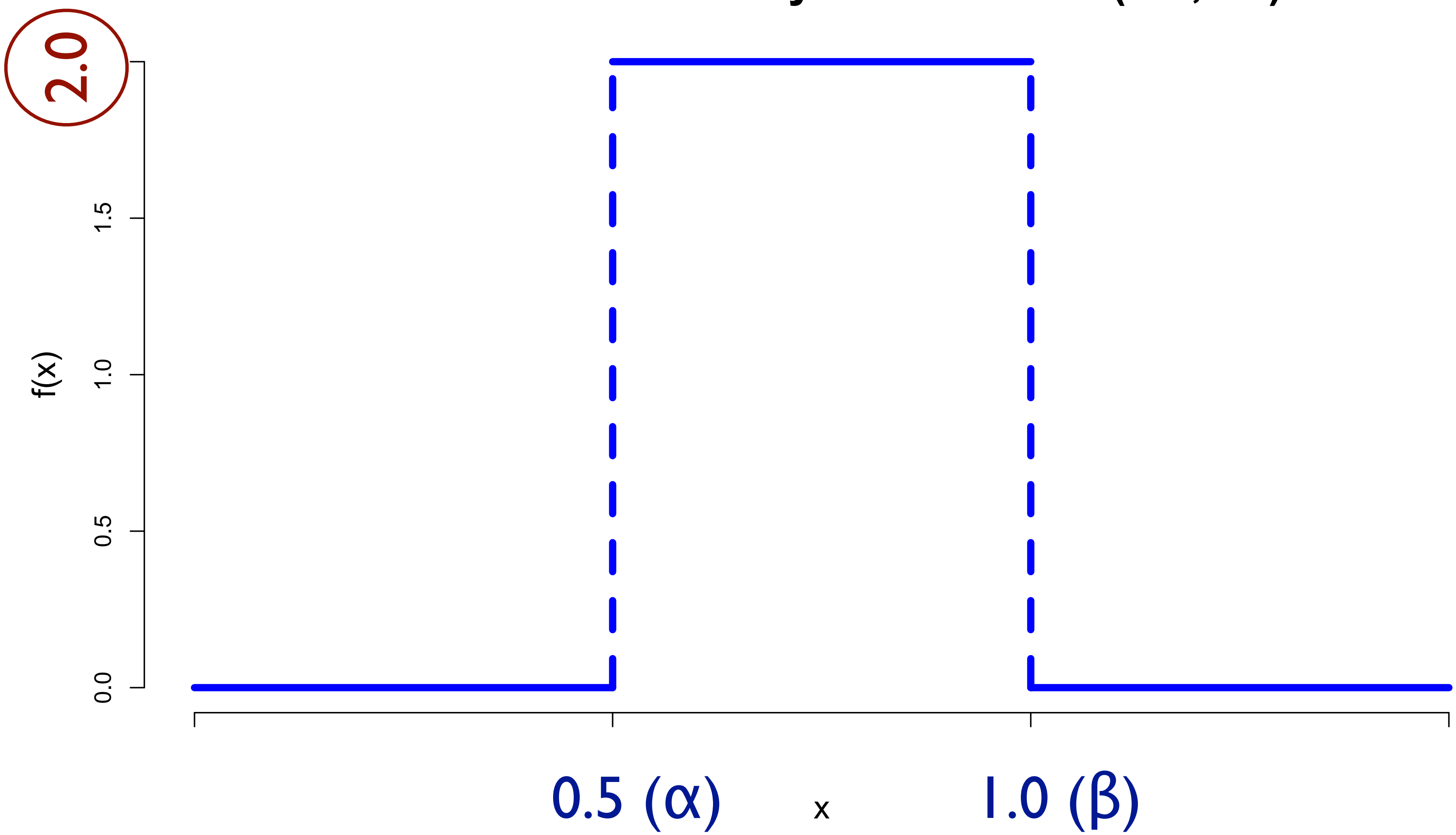
$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx$$

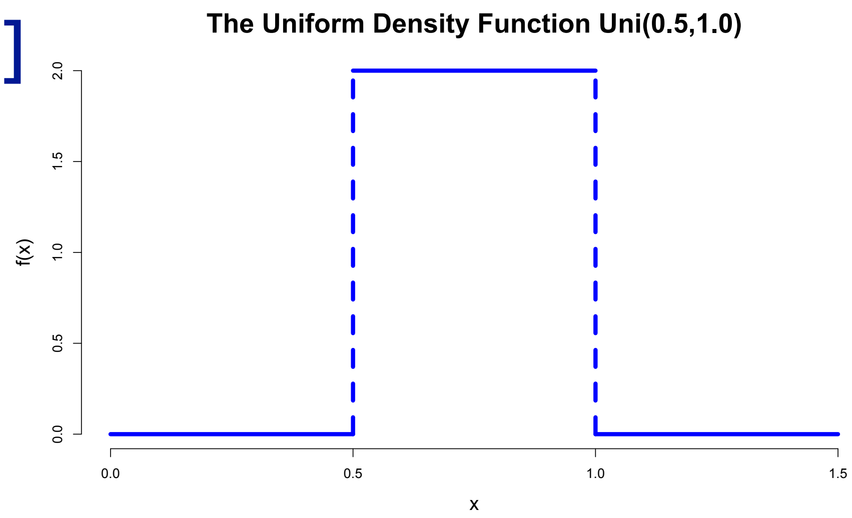
uniform random variables

X ~ Uni(α,β) is uniform in [α,β] $f(x) = \begin{cases} \frac{1}{\beta - \alpha} & x \in [\alpha, \beta] \\ 0 & \text{otherwise} \end{cases}$

The Uniform Density Function Uni(0.5,1.0)



X ~ Uni(α,β) is uniform in [α,β]



$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & x \in [\alpha, \beta] \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr(a \leq X \leq b) = \int_a^b f(x)\,dx = \frac{b-a}{\beta-\alpha}$$

if α≤a≤b≤β:

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \frac{\alpha+\beta}{2}$$

Yes, you should review your basic calculus; e.g., these 2 integrals would be good practice.

uniform random variable: example

X ~ Uni(α,β) is uniform in [α,β]

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & x \in [\alpha, \beta] \\ 0 & \text{otherwise} \end{cases}$$

You want to read a disk sector from a 7200rpm disk drive. Let T be the time you wait, in milliseconds, after the disk head is positioned over the correct track, until the desired sector rotates under the head.

T ~ Uni(0, 8.33)

Average Wait? 4.17ms

Radioactive decay: How long until the next alpha particle?

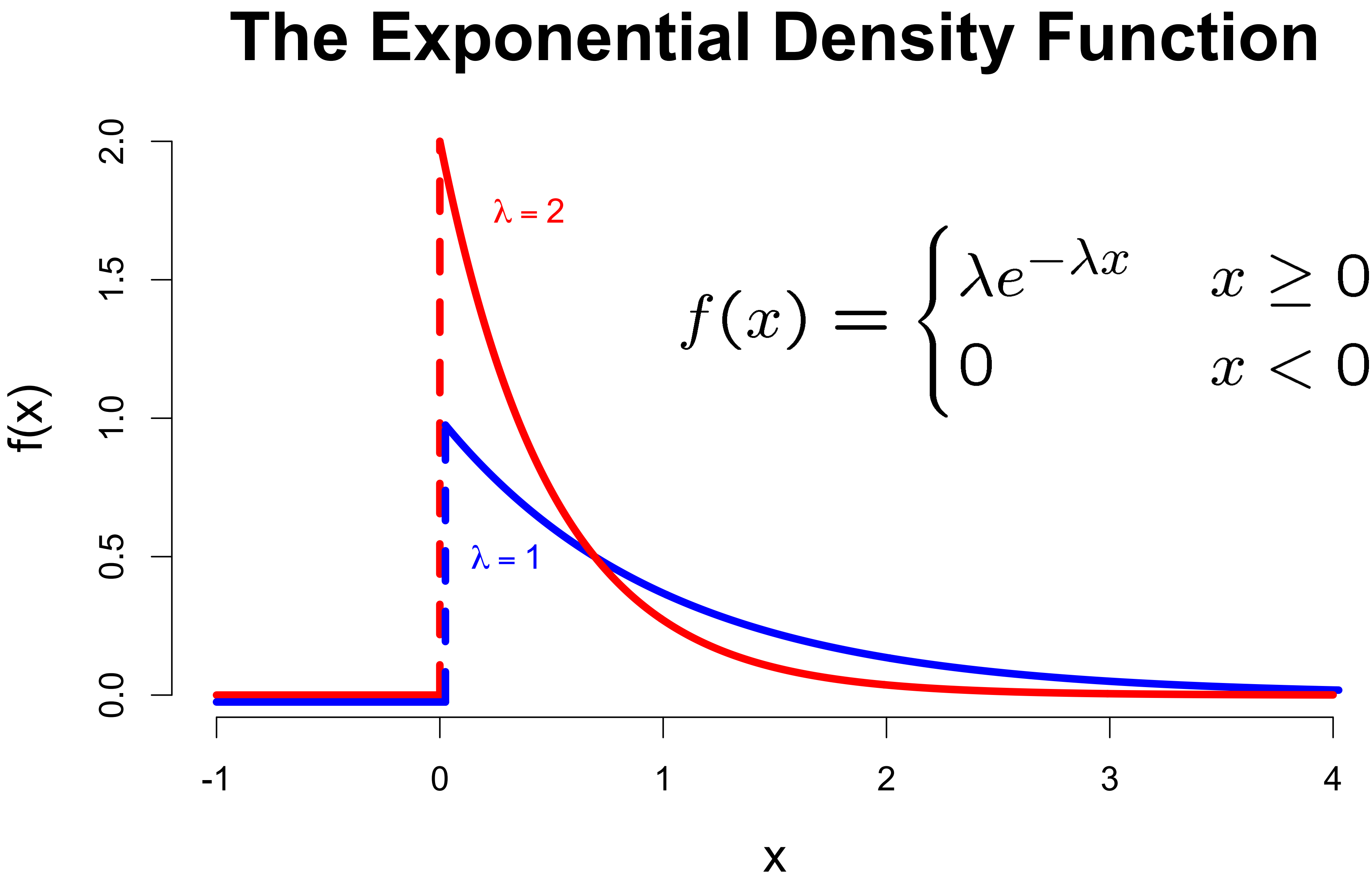
Customers: how long until the next customer/packet arrives at the checkout stand/server?

Buses: How long until the next #71 bus arrives on the Ave?

- Yes, they have a schedule, but given the vagaries of traffic, riders with-bikes-and-baby-carriages, etc., can they stick to it?

Assuming events are independent, happening at some fixed average rate of λ per unit time – the waiting time until the next event is exponentially distributed (next slide)

X ~ Exp(λ)



The Exponential Density Function
λ = 2
λ = 1
$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$
f(x)
x
-1
0
1
2
3
4
0.0
0.5
1.0
1.5
2.0

exponential random variables

X ~ Exp(λ)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$E[X] = \frac{1}{\lambda} \qquad \mathrm{Var}[X] = \frac{1}{\lambda^2}$$

$$\Pr(X \geq t) = e^{-\lambda t} = 1\text{-}F(t)$$

Memorylessness:

$$\Pr(X > s + t \mid X > s) = \Pr(X > t)$$

Assuming exp distr, if you've waited *s* minutes, prob of waiting *t* more is exactly same as *s = 0*

Gambler's fallacy: "I'm due for a win"

Relation to the Poisson: same process, different measures:

Poisson: *how many* events in a *fixed time*;
Exponential: *how long* until the *next event*

λ is avg # per unit time;
$1/\lambda$ is mean wait

Relation to geometric: Geometric is discrete analog:

How long to a Head, 1 flip per sec, prob p vs
How long to a Head, 2 flips per sec, prob p/2, vs
How long to a Head, 3 flips per sec, prob p/3, vs
⋮

All have same mean

Limit is exponential with parameter 1/p

see also B&T fig 3.8

A brief message from the Math SuperPAC

(This message not approved by *any* political candidate ...)

KCTS 9 Washington Poll

KCTS9 | W

Governor

If the election for Governor of Washington were held today, would you vote for **(ROTATE NAMES) Jay Inslee**, who prefers the Democratic Party, or **Rob McKenna,** who prefers the Republican Party?

	Registered Voters	Likely Voters
Inslee – certain	39.5%	41.9%
Inslee – could change	5.4%	4.4%
Undecided – lean Inslee	2.3%	2.4%
Undecided	7.4%	5.8%
Undecided – lean McKenna	1.8%	1.0%
McKenna – could change	3.7%	3.6%
McKenna – certain	40.0%	41.0%
Total – Inslee	**47.2%**	**48.7%**
Total – McKenna	**45.5%**	**45.6%**

632 likely voters: +/- 3.9%

722 registered voters: +/- 3.6%; 632 likely voters: +/- 3.9%, Oct 18-31, 2012

KCTS9.org/vote2012

http://kcts9.org/sites/default/files/kcts9wapoll_oct31.pdf we'll see this more formally later

Many registered voters

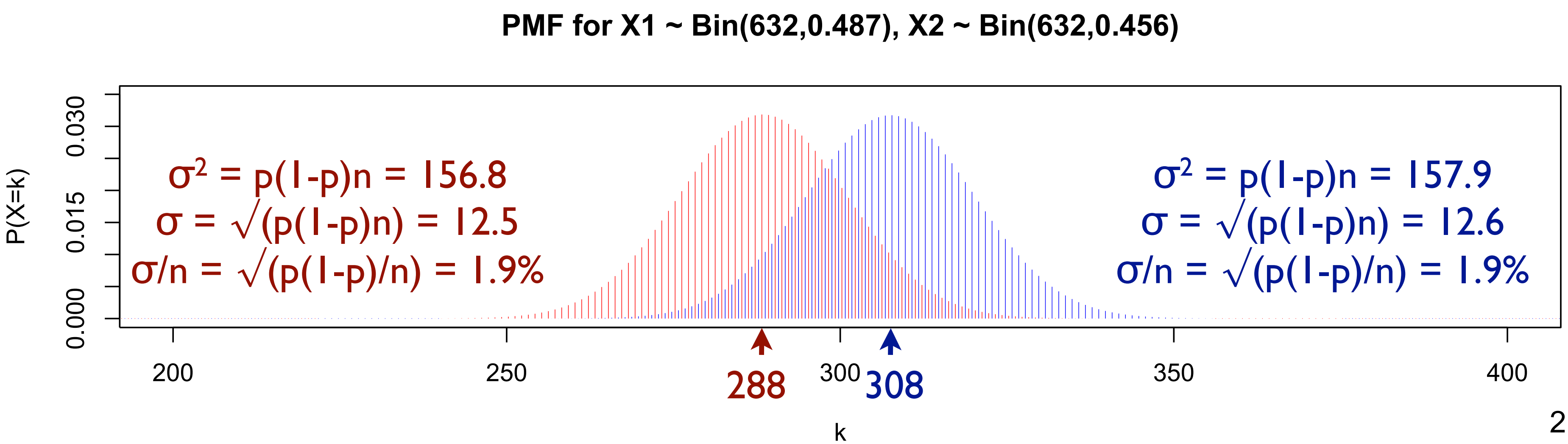
Suppose a fraction p of them will vote for Inslee

Call 632 of them at random, ask who they like

Suppose 48.7% (308) say "Inslee," [& 45.6% (288) McKenna]

Binomial random variable, mean pn, variance $\sigma^2 = p(1-p)n$

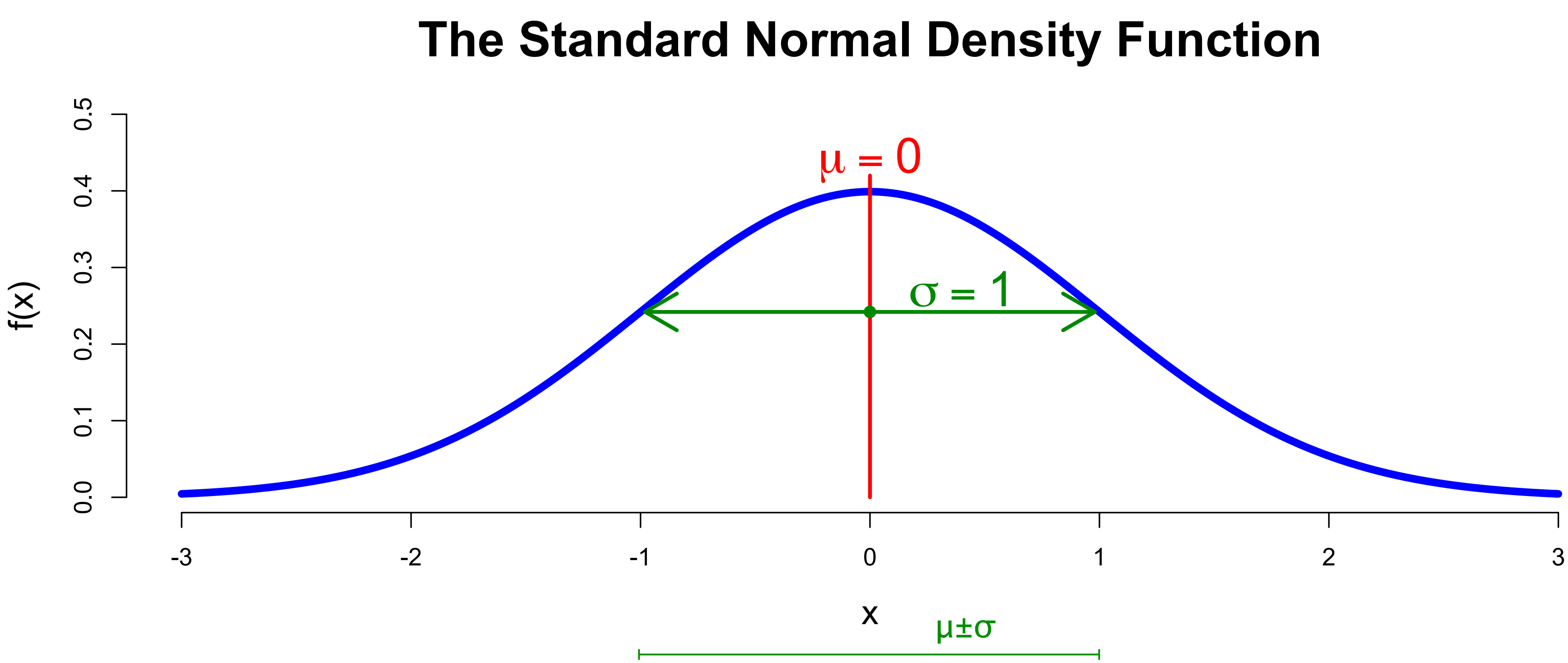
If the gap between M & I is greater than, say, 2σ, we can be reasonably sure the poll difference is "real," but prediction is sketchy if the gap is smaller. I.e., "margin of error" is ~ 2σ



X is a normal (aka Gaussian) random variable $X \sim N(\mu, \sigma^2)$

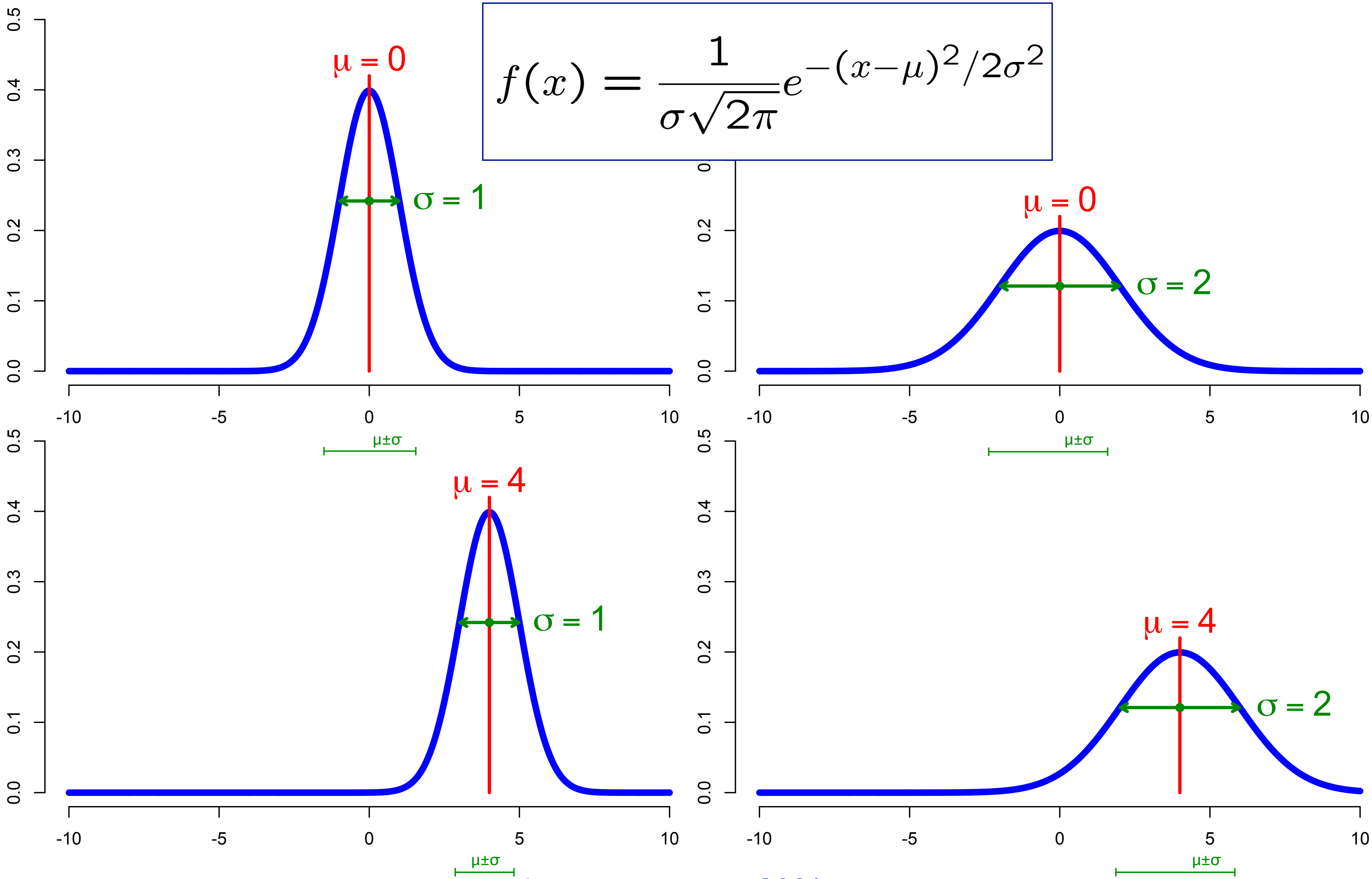
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$E[X] = \mu \qquad \mathrm{Var}[X] = \sigma^2$$



changing μ, σ

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$



density at μ is ≈ .399/σ

X is a normal random variable $X \sim N(\mu,\sigma^2)$

$Y = aX + b$

$E[Y] = E[aX+b] = a\mu + b$

$Var[Y] = Var[aX+b] = a^2\sigma^2$

$Y \sim N(a\mu + b, a^2\sigma^2)$

E[ ], Var[ ] as expected;
"normality" is the surprise

Important special case: $Z = (X-\mu)/\sigma \sim N(0,1)$

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$

$Z \sim N(0,1)$ "*standard* (or *unit*) normal"

Use $\Phi(z)$ to denote CDF, i.e.

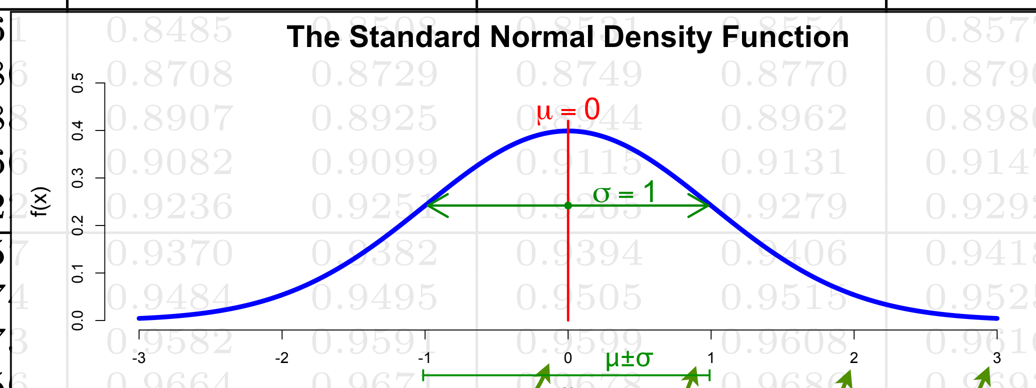
$$\Phi(z) = \Pr(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$$

no closed form ☹

Table of the Standard Normal Cumulative Distribution Function $\Phi(Z)$

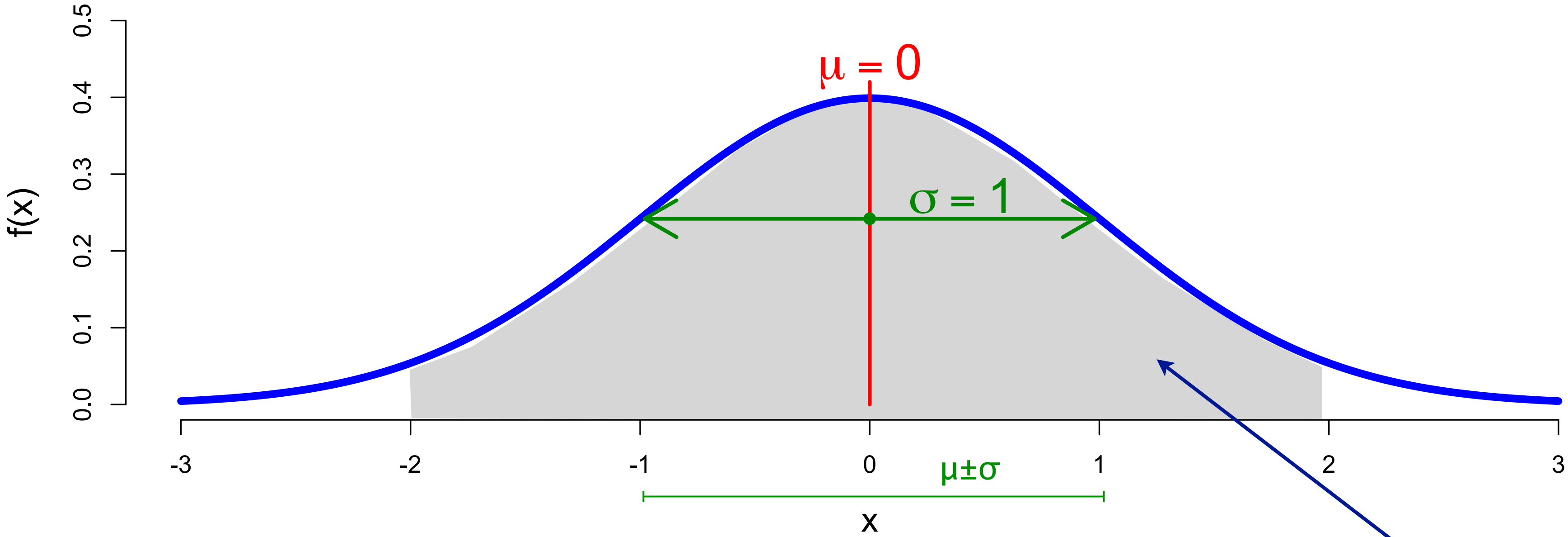
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	[illegible]	0.7517
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	[illegible]	0.7823
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	[illegible]	[illegible]	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	[illegible]	[illegible]	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997

$\Phi(.46)$



E.g., see B&T p155, p531

The Standard Normal Density Function



If $Z \sim N(\mu,\sigma^2)$ what is $P(\ \mu-\sigma < Z < \mu+\sigma\ )$?

$P(\ \mu - \sigma < Z < \mu + \sigma\ ) = \Phi(1) - \Phi(-1) \approx 68\%$

$P(\ \mu - 2\sigma < Z < \mu + 2\sigma\ ) = \Phi(2) - \Phi(-2) \approx 95\%$

$P(\ \mu - 3\sigma < Z < \mu + 3\sigma\ ) = \Phi(3) - \Phi(-3) \approx 99\%$

Why?

$\mu - k\sigma < \underbrace{\boxed{Z}}_{N(\mu,\sigma^2)} < \mu + k\sigma \quad \Leftrightarrow \quad -k < \underbrace{\boxed{(Z-\mu)/\sigma}}_{N(0,1)} < +k$

X ~ Bin(n,p)   E[X] = np   Var[X] = np(1-p)

Poisson approx: good for n large, p small (np constant)

Normal approx: For large n, (p stays fixed):

X ≈ Y ~ N(E[X],Var[X]) = N(np,np(1-p))

Normal approximation good when np(1-p) ≥ 10

DeMoivre-Laplace Theorem:

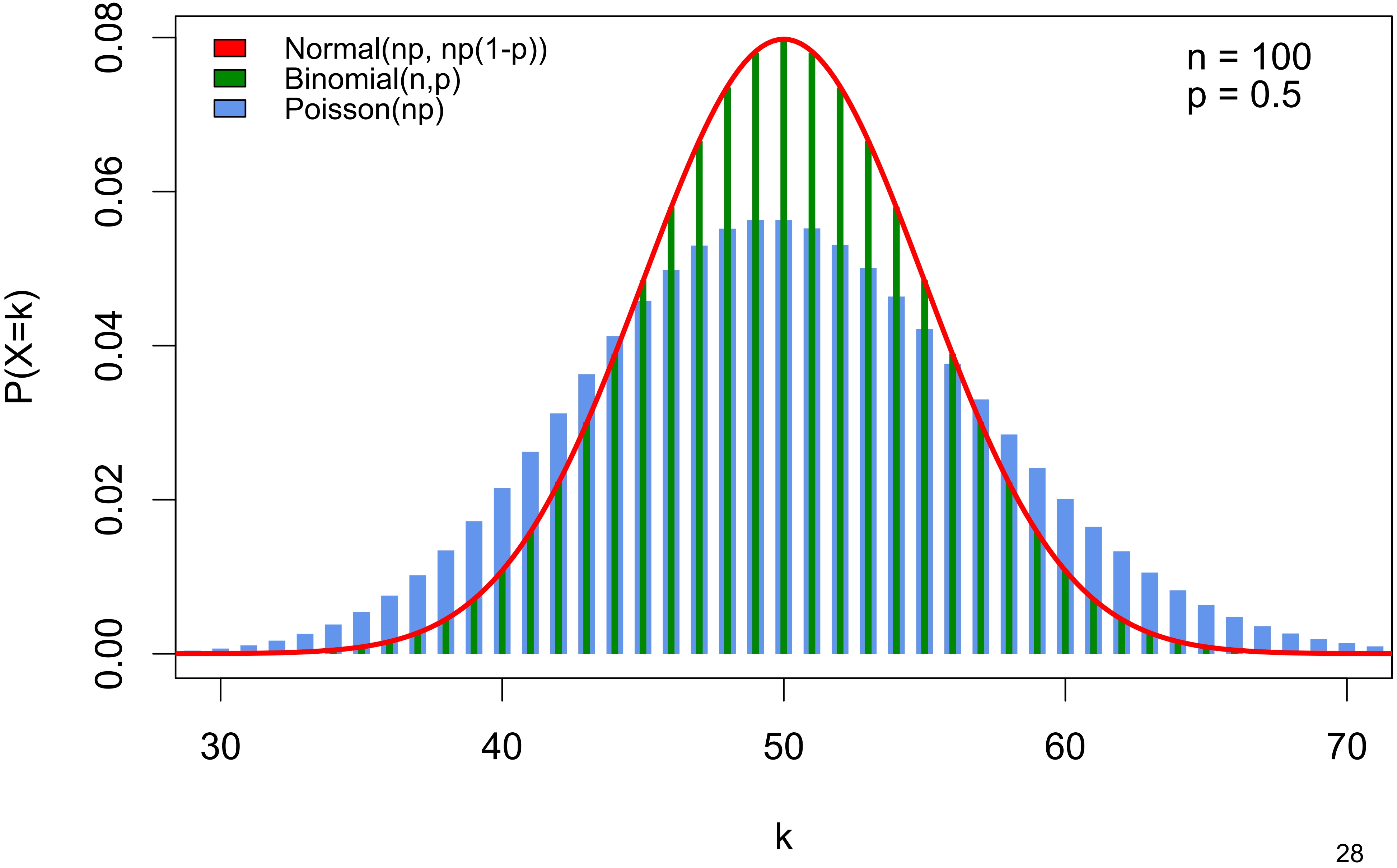
Let S_n = number of successes in n trials (with prob. p).

Then, as n→∞:

$$Pr\left(a \leq \frac{S_n - np}{\sqrt{np(1-p)}} \leq b\right) \longrightarrow \Phi(b) - \Phi(a)$$

Equivalently:

$$Pr(a \leq S_n \leq b) \longrightarrow \Phi\left(\frac{b-np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{a-np}{\sqrt{np(1-p)}}\right)$$



Normal(np, np(1-p))
Binomial(n,p)
Poisson(np)
n = 100
p = 0.5
P(X=k)
0.00
0.02
0.04
0.06
0.08
30
40
50
60
70
k

DeMoivre-Laplace and the "continuity correction"

Potential pitfalls: Let S = # heads in100 flips of a fair coin

$$Pr(a \leq S \leq b) \longrightarrow \Phi\left(\frac{b-50}{5}\right) - \Phi\left(\frac{a-50}{5}\right)$$

i) $Pr(50 \leq S \leq 50) \approx .08$, but $\Phi(0) - \Phi(0) = 0$

ii) $Pr(50.01 \leq S \leq 50.99) = 0$, but $\Phi(.99/5) - \Phi(.01/5) \approx .08$

The "continuity correction":

Imagine *discretizing* the normal density by shifting probability mass at non-integer x to the nearest integer (i.e., "rounding" x). Then the probability of S falling in the (*integer*) interval [a, ..., b], inclusive, is ≈ the probability of a normal r.v. with the same μ,σ^2 falling in the (*real*) interval [a-½ , b+½].

E.g. i) $Pr(50 \leq S \leq 50) = Pr(49.5 \leq S \leq 50.5) \approx \Phi(-0.1) - \Phi(0.1) \approx .08$

ii) $Pr(50.01 \leq S \leq 50.99)$ = Pr(the empty set of integers) = 0

More: Feller, 1945 http://www.jstor.org/stable/10.2307/2236142

Ex: Fair coin flipped 40 times. Probability of 20 or 21 heads?

Exact answer:

$$P(X = 20 \vee X = 21) \quad = \quad \left[\binom{40}{20} + \binom{40}{21} \right] \left(\frac{1}{2} \right)^{40} \approx \boxed{0.2448}$$

Normal approximation:

$$\begin{aligned} P(20 \le X < 22) &= P(19.5 \le X \le 21.5) \\ &= P\left(\frac{19.5 - 20}{\sqrt{10}} \le \frac{X - 20}{\sqrt{10}} \le \frac{21.5 - 20}{\sqrt{10}} \right) \\ &\approx P\left(-0.16 \le \frac{X - 20}{\sqrt{10}} \le 0.47 \right) \\ &\approx \Phi(0.47) - \Phi(-0.16) \approx \boxed{0.2452} \end{aligned}$$

{19.5 ≤ X ≤ 21.5} is the set of *reals* that round to the set of *integers* in {20 ≤ X < 22}

Dialog in class:

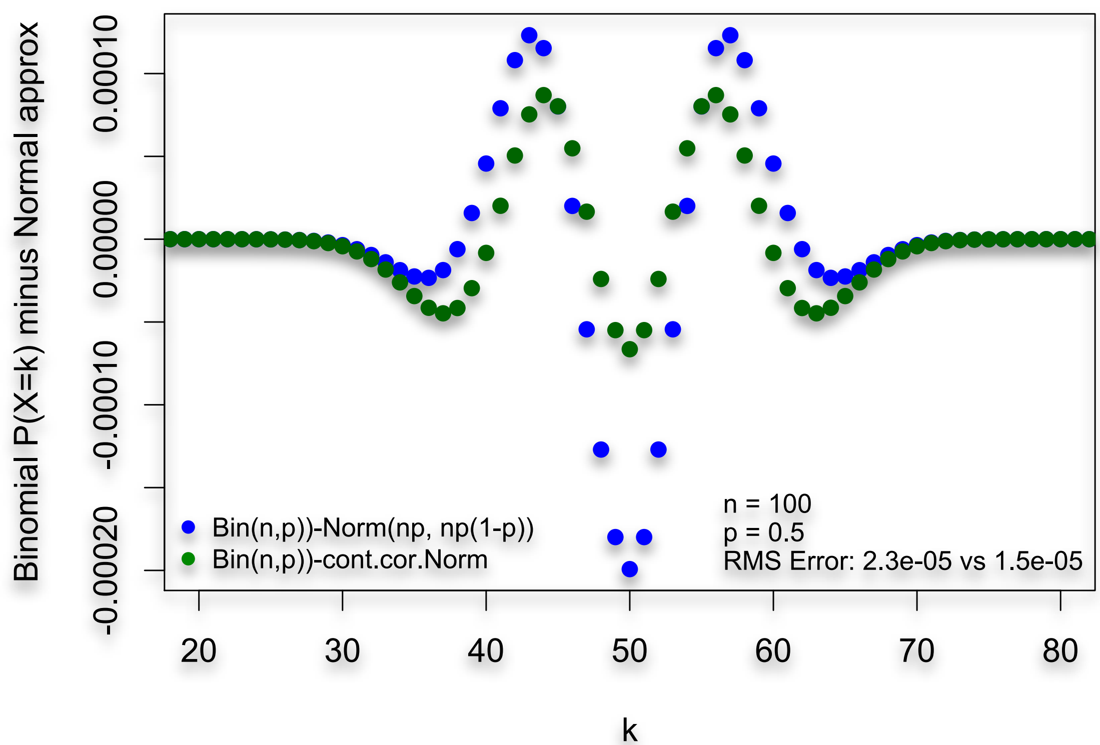
Q (Student): "Why add/subtract .5? Why not, say, .25?"

A (Prof Evil): "For integer X, the area under the normal density in the strip X±½ is approximately the probability of sampling a normal r.v. that rounds to X, but the area in the strip X±¼ is only about half that."

Q: "What about doubling that area, would that be better?"

A: "Hmm, I dunno, but extrapolating, you could also look at 1/ε times the area in the X±ε/2 strip, which in the limit is the density at X."

Graph compares ±½ version (green) to density (blue). ±½ is better on average, but not uniformly better.



Consider i.i.d. (independent, identically distributed) random vars $X_1, X_2, X_3, \ldots$

X_i has $\mu = E[X_i]$ and $\sigma^2 = Var[X_i]$

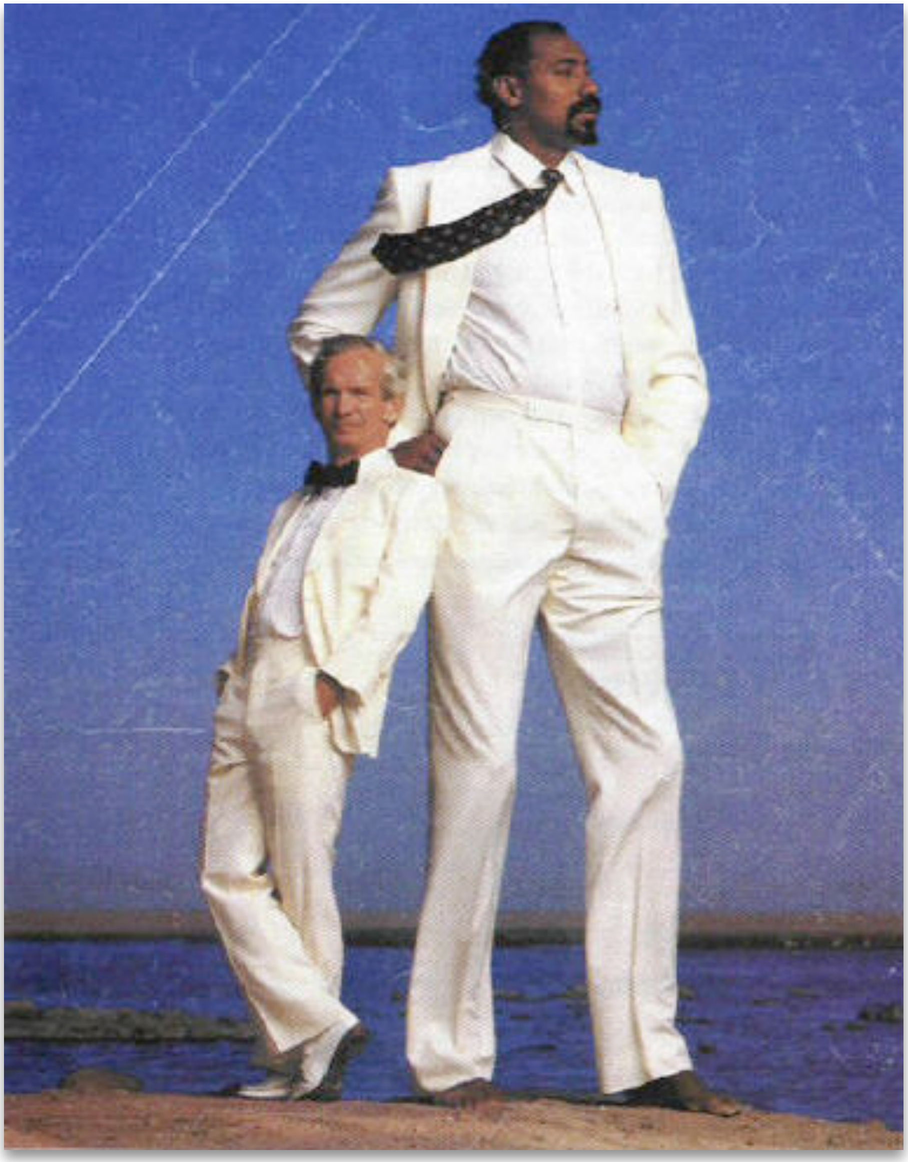
As $n \to \infty$,

$$\frac{X_1 + X_2 + \cdots + X_n - n\mu}{\sigma\sqrt{n}} \longrightarrow N(0,1)$$

Restated: As $n \to \infty$,

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

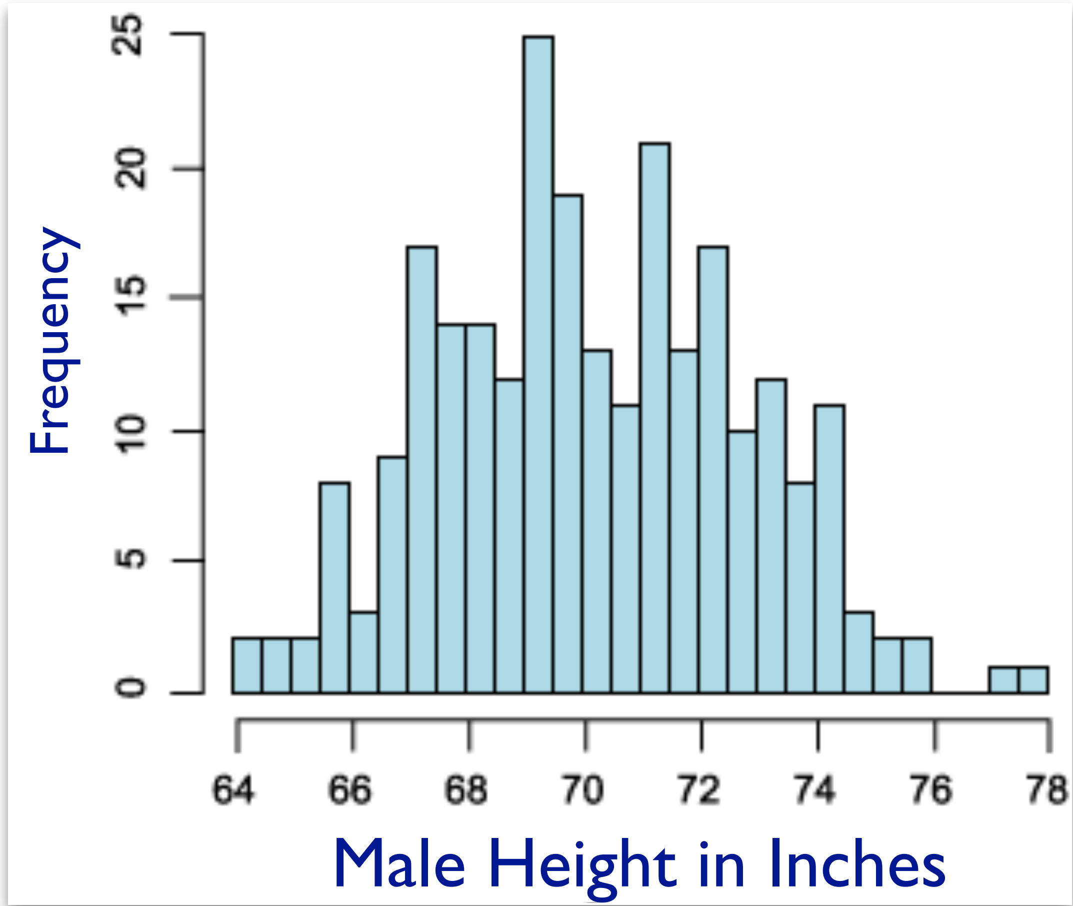
How tall are you? Why?



Credit: Annie Leibovitz, © 1987 ?

Willie Shoemaker & Wilt Chamberlain

Human height is approximately normal.



Why might that be true?

R.A. Fisher (1918) noted it would follow from CLT if height were the sum of many independent random effects, e.g. many genetic factors (plus some environmental ones like diet). I.e., suggested part of *mechanism* by looking at *shape* of the curve. (WAY before anyone really knew what genes, DNA, etc. were...)

Meta-analysis of Dense Genecentric Association Studies Reveals Common and Uncommon Variants Associated with Height, Lanktree, et[194] al.

The American Journal of Human Genetics **88**, 6–18, January 7, 2011

Table 1. Sixty-Four **Loci Showing Significant Evidence for Association with Adult Height, Identified with the Use of the IBC Array**

(and hundreds more probably exist)

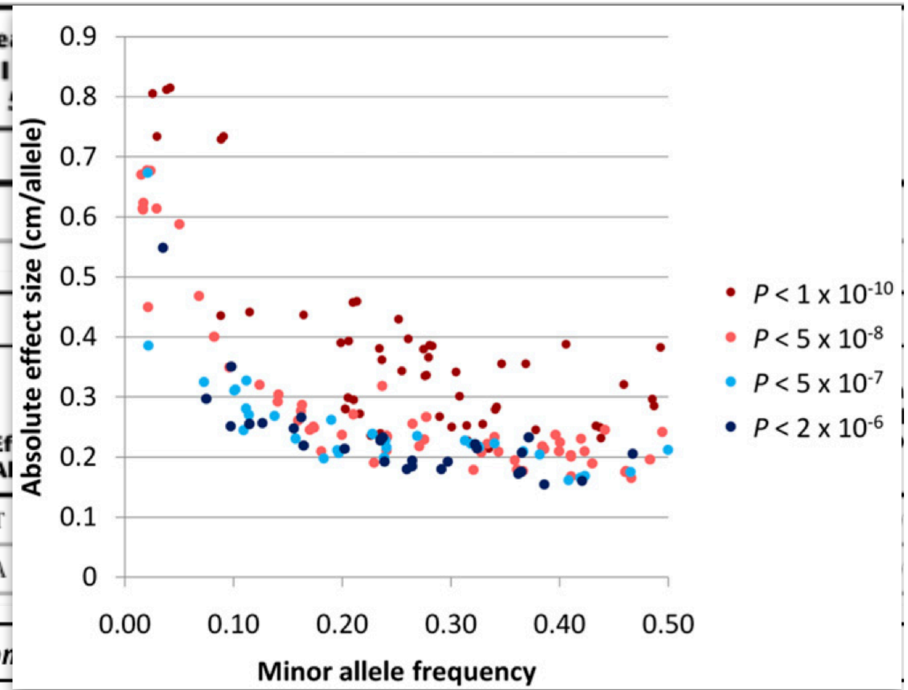
Locus Rank	Chr.	Candidate Gene[a]	SNP[a]	Effect Allele	MAF	European Phase I (up to 53,394) Effect
1	7q22	*CDK6*	rs4272	A	0.21	−0.46
2	6p21	*HMGA1*	[illegible]	[illegible]	[illegible]	[illegible]
3	12q15	*HMGA2*				
4	20q11	*MMP24*				
5	17q23	*MAP3K3*				
6	17q24	*GH1-GH2*				
7	1p36	*MFAP2*				
8	15q26	*IGF1R*				
9	7p22	*GNA12*				
10	17q23	*TBX2*				
11	12q22	*SOCS2*				
12	9q22	*PTCH1*				
13	14q11	*NFATC4*				
14	15q26	*ACAN*				
15	2q24	*NPPC*				
16	6p21	*PPARD*				
17	20q11	*MYH7B*				
18	19q13	*IL11*				

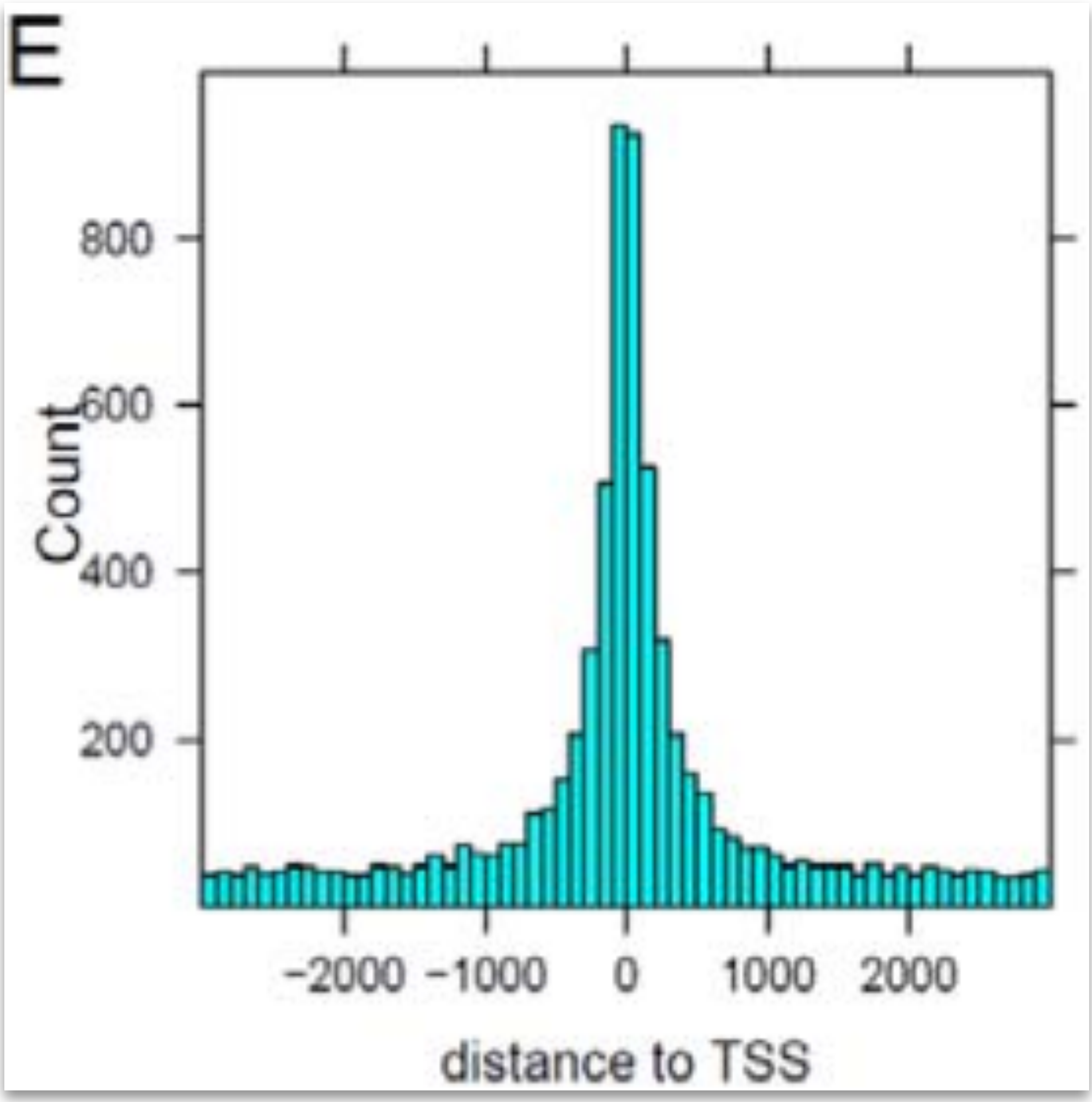
Table 1. *Continued*

Locus Rank	Chr.	Candidate Gene[a]	SNP[a]	Effect Allele
28	2p23	*GCKR*	rs780094	T
29	1q41	*TGFB2*	rs900	A
30	20q11	*CDK5RAP1*		
31	2p12	*EIF2AK3*		
32	19p13	*INSR*		
33	6q25	*ESR1*		
34	2q37	*DIS3L2*		
35	2q35	*PLCD4*		
36	1p36	*RPS6KA1*		
37	15q21	*CYP19A1*		
38	5q31	*SLC22A5*		
39	7p15	*JAZF1*		
40	17p13	*POLR2A*		
41	1p22	*PKN2*		
42	7q22	*CNOT4*		

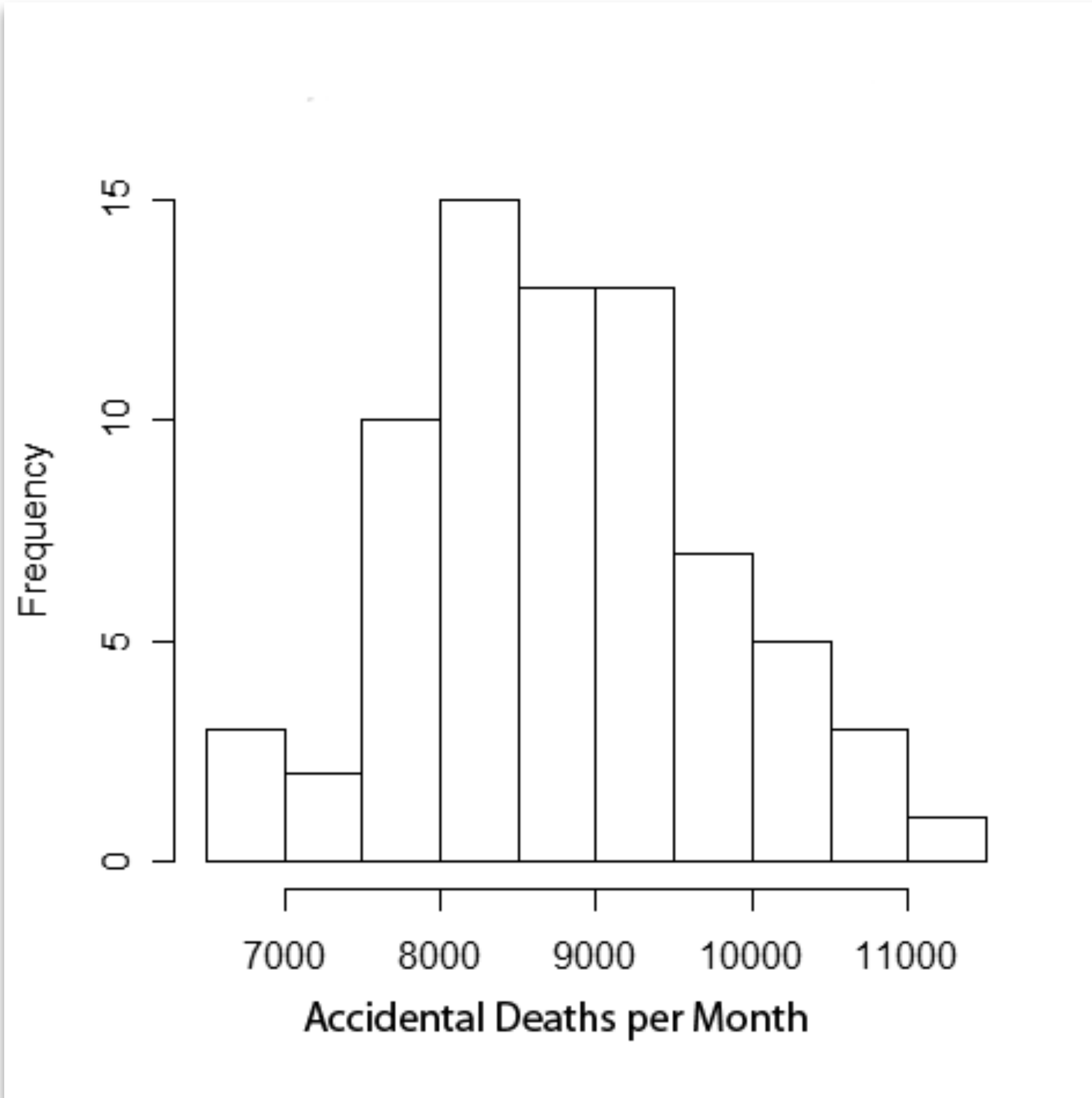
Table 1. *Continued*

Locus Rank	Chr.	Candidate Gene[a]	SNP[a]	Effect Allele	MAF	European Ancestry Phase I (up to 53,394) Effect	p	I²
54	1p22	*COL24A1*	rs2046159	A	0.16	0.23	3.8×10^{-5}	2
55	1q23	*DUSP23*	rs1129923	A	0.10	−0.25	2.7×10^{-4}	0
56	10q22	*MAT1A*	rs7087728	A	0.18	0.22	2.2×10^{-4}	0
57	2p15	*PPP3R1*	rs1822469	T	0.41	−0.14	7.8×10^{-4}	9
58	7q36	*ATG9B*	rs1800783	A	0.38	−0.16	2.0×10^{-4}	0
59	14q11	*BCL2L2*	rs3210043	A	0.16	0.25	9.7×10^{-6}	0
60	4p14	*RFC1*	rs11096991	T	0.35	0.15	3.6×10^{-4}	0
61	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

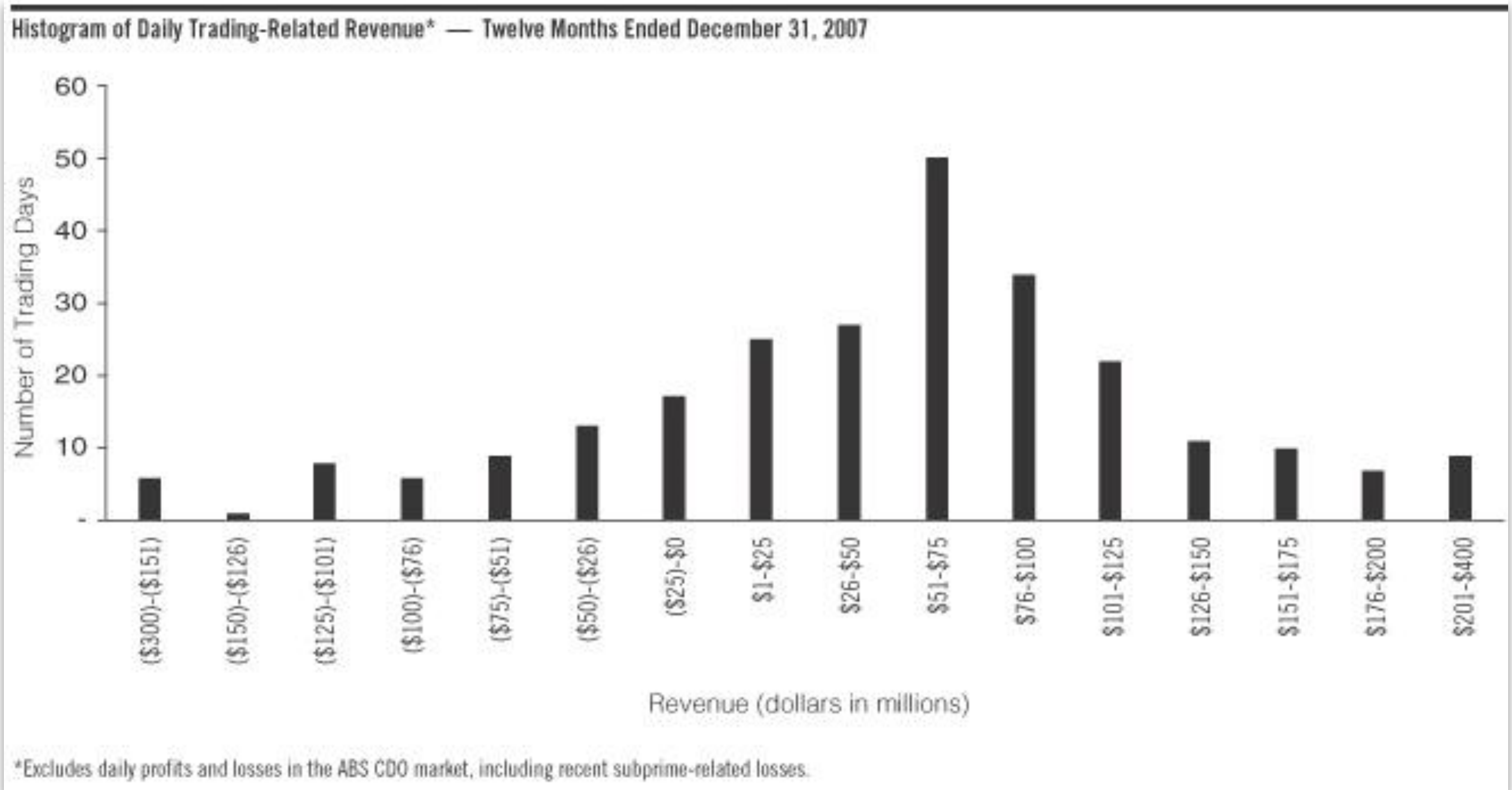



E
Count
800
600
400
200
−2000
−1000
0
1000
2000
distance to TSS

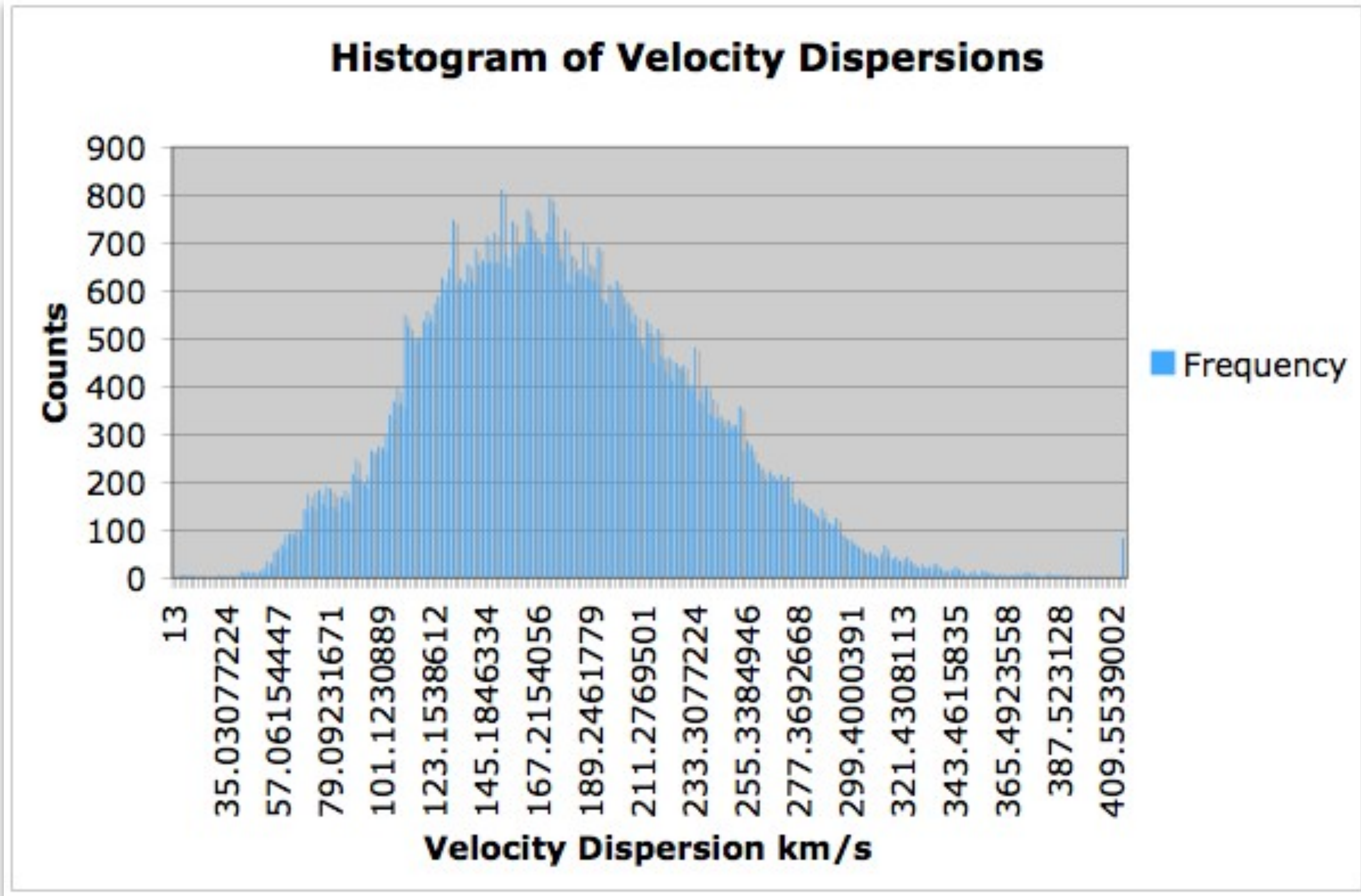
in the real world…



in the real world…



in the real world…



pdf and cdf

$f(x) = \frac{d}{dx} F(x)$ $\qquad$ $F(a) = \int_{-\infty}^{a} f(x)\, dx$

sums become integrals, e.g.

$E[X] = \Sigma_x \, xp(x)$ $\qquad$ $E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

most familiar properties still hold, e.g.

$E[aX+bY+c] = aE[X]+bE[Y]+c$

$Var[X] = E[X^2] - (E[X])^2$

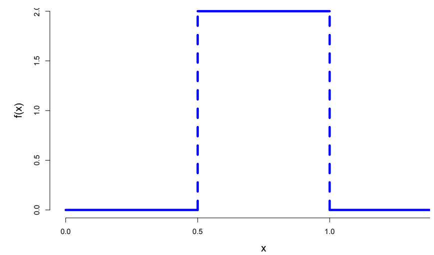
Three important examples

X ~ Uni(α,β) uniform in [α,β]

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & x \in [\alpha, \beta] \\ 0 & \text{otherwise} \end{cases}$$

$E[X] = (\alpha+\beta)/2$

$\mathrm{Var}[X] = (\alpha-\beta)^2/12$

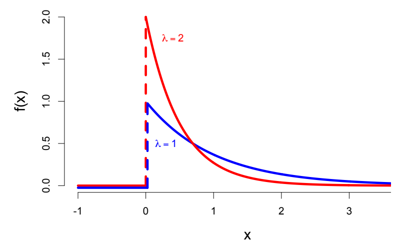


X ~ Exp(λ) exponential

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$E[X] = \frac{1}{\lambda}$$

$$\mathrm{Var}[X] = \frac{1}{\lambda^2}$$



X ~ N(μ, σ²) normal (aka Gaussian, aka the big Kahuna)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$E[X] = \mu$$

$$\mathrm{Var}[X] = \sigma^2$$