

Midterm Review

coverage

everything in text chapters 1-2, slides & homework pre-exam except “continuous random variables” started this week is included, except as noted below.

mechanics

1 page of notes; closed book

I’m more interested in setup and method than in numerical answers, so concentrate on giving a clear approach, perhaps including a terse English outline of your reasoning.

Corollary: calculators are probably irrelevant, but bring one to the exam if you want, just in case.

chapter 1: combinatorial analysis

counting principle (product rule)

permutations

combinations

indistinguishable objects

binomial coefficients

binomial theorem

partitions & multinomial coefficients

inclusion/exclusion

pigeon hole principle

chapter 1: axioms of probability

sample spaces & events

axioms

complements, Venn diagrams, deMorgan,
mutually exclusive events, etc.

equally likely outcomes

chapter 1: conditional probability and independence

conditional probability

chain rule, aka multiplication rule

total probability theorem

bayes rule *yes, learn the formula*

odds

independence

conditional independence

gambler's ruin

discrete random variables

probability mass function (pmf)

expectation of X

expectation of $g(X)$ (i.e., a function of an r.v.)

linearity: expectation of $X+Y$ and $aX+b$

variance

cumulative distribution function (cdf)

cdf as sum of pmf from $-\infty$

joint and marginal distributions

important examples:

know pmf, mean, variance of these

bernoulli, binomial, poisson, geometric, uniform

some important (discrete) distributions

Name	PMF	$E[k]$	$E[k^2]$	σ^2
Bernoulli(p)	$f(k) = \begin{cases} 1-p & \text{if } k=0 \\ p & \text{if } k=1 \end{cases}$	p	p	$p(1-p)$
Binomial(p, n)	$f(k) = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, 1, \dots, N$	np		$np(1-p)$
Poisson(λ)	$f(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, \dots$	λ	$\lambda(\lambda + 1)$	λ
Geometric(p)	$f(k) = p(1-p)^{k-1}, k = 0, 1, \dots$	$1/p$	$(2-p)/p^2$	$(1-p)/p^2$
Hypergeometric(n, N, m)	$f(k) = \frac{\binom{m}{k} \binom{N-m}{n-k}}{\binom{N}{n}}, k = 0, 1, \dots, N$	nm/N	$\frac{nm}{N} \left(\frac{(n-1)(m-1)}{N-1} + 1 \right)$	$\frac{nm}{N} \left(\frac{(n-1)(m-1)}{N-1} + 1 - \frac{nm}{N} \right)$

Calculus is a prereq, but I'd suggest the most important parts to brush up on are:

taylor's series for e^x

sum of geometric series: $\sum_{i \geq 0} x^i = 1/(1-x)$ ($0 \leq x < 1$)

Tip: multiply both sides by $(1-x)$

$$\sum_{i \geq 1} ix^{i-1} = 1/(1-x)^2$$

Tip1: slide numbered 32 in "random variables" lecture notes, or text

Tip2: if it were $\sum_{i \geq 1} ix^{i+1}$, say, you could convert to the above form by dividing by x^2 etc.; 1st few terms may be exceptions

integrals & derivatives of polynomials, e^x ;

chain rule for derivatives; integration by parts

Good Luck!