Quiz Section 2: Proofs

Task 1 – Simple Formal Proofs

a) Given $(q \wedge r), (r \rightarrow \neg s)$ and $(s \vee p)$, show that p holds.

b) Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

c) Given $\neg(\neg r \vee k)$, $\neg q \vee \neg s$ and $(p \rightarrow q) \wedge (r \rightarrow s)$, show that $\neg p$ holds.

Task 2 – Direct Proofs

a) Show that $\neg k \rightarrow s$ follows from $k \vee q$, $q \rightarrow r$ and $r \rightarrow s$ with a formal proof.

b) Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

Task 3 – Predicate Logic Proofs

a) Given $\forall x\,(T(x) \rightarrow M(x))$, we wish to prove $(\exists x\,T(x)) \rightarrow (\exists y\,M(y))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x\,(T(x) \rightarrow M(x))$ ____________________

    2.1. $\exists x\,T(x)$ ____________________
    2.2. $T(c)$ ____________________
    2.3. $T(c) \rightarrow M(c)$ ____________________
    2.4. $M(c)$ ____________________
    2.5. $\exists y\,M(y)$ ____________________

2. $(\exists x\,T(x)) \rightarrow (\exists y\,M(y))$ ____________________

b) Given $\forall x\, \exists y\, (T(x) \to S(y,x))$, we wish to prove $\exists x\, (T(x) \to \forall y\, S(y,x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\exists x\, (T(x) \to \forall y\, S(y,x))$ ____________________
2. $T(c) \to \forall y\, S(y,c)$ ____________________

> Let a be arbitrary.
>
> > 3.1.1. $T(c)$ ____________________
> >
> > 3.1.2. $\forall y\, S(y,c)$ ____________________
> >
> > 3.1.3. $S(a,c)$ ____________________
>
> 3.1. $T(c) \to S(a,c)$ ____________________
>
> 3.2. $\exists x\, (T(x) \to S(a,x))$ ____________________

3. $\forall y\, \exists x\, (T(x) \to S(y,x))$ ____________________

Task 4 – English Proofs

Let domain of discourse be the integers. Consider the following claim:

$$\forall x\, \forall y\, ((\text{Even}(x) \land \text{Odd}(y)) \to \text{Odd}(x+y))$$

In English, this says that, for any even integer x and odd integer y, the integer $x+y$ is odd.

a) Write a **formal proof** that the claim holds.

b) Translate your formal proof to an **English proof**.

Keep in mind that your proof will be read by a *human*, not a computer, so you should explain the algebra steps in more detail, whereas some of the predicate logic steps (e.g., Elim $\exists$) can be skipped.